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# A THEORETICAL MODEL FOR CAVE AIR FLOW: THE CHIMNEY EFFECT

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "A Theoretical Model for Cave Air Flow: The Chimney Effect" submitted by Richard Edward Baldwin in partial fulfilment of the requirements for the degree of Master of Science.

# ABSTRACT

Cave systems with two or more entrances at different elevations often exhibit seasonally directional air flow. This flow is analogous to that in a chimney and is thus referred to as the "chimney" or "stack" effect. Previous attempts to model cave winds of this type have greatly simplified the phenomenon by assuming <u>static</u> conditions and then estimating air pressure or density differences between the cave atmosphere and the outside air. This approach is basically incorrect when applied to dynamic conditions, and may lead to substantial errors in estimating speleomicroclimatic parameters. Wigley and Brown (1972) have predicted temperature profiles under dynamic conditions, and this thesis extends their work to a general model for this type of air flow. The assumption of the incompressibility of air is analysed and found to be a reasonable simplification. The results are of significance to geomorphologists investigating inaccessible karst systems, biospeleologists concerned with the distribution of life in caves, and mine ventilation engineers.

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TABLE OF CONTENTS

LIST OF TABLES

## LIST OF FIGURES

# I - CAVES AND THE CAVE ATMOSPHERE

From very early in human prehistory, man has been associated with caves. They have provided him with shelter from the elements, protection from his enemies, and often a means of living by the extraction of valuable minerals. Even in the heavily industrialised western world, they are still used as storage areas for perishables such as wine, and as a source of bat guano, an excellent organic fertilizer. More recently, however, caves have come under the scrutiny of the scientific community. Aside from the obvious concerns of how caves are formed and what effect they have on local and regional drainage patterns, an aspect of speleology that has elicited much interest is speleomicroclimatology, the study of the cave atmosphere. Of considerable interest is the relative invariability of cave atmospheres, since this invariability is one of the most useful features of caves. Because internal temperatures approximate the mean annual temperature outside a cave,[1] caves which are located in regions of significant seasonal temperature variations are substantially warmer than the exterior region in winter and substantially cooler in summer. As a means of surviving the harsh winters of early post-glacial times, for example, caves were nearly ideal.

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[1] As will be seen later, internal temperatures may vary significantly near entrances which draw air, but the amplitude of these fluctuations decreases away from an entrance to the point where temperatures are nearly invariant.

The cave atmosphere also determines, to a large extent, the flora and fauna found beneath the surface. Poulson and White (1969) have characterised caves as ecological and evolutionary laboratories where relations between surface and subsurface species, as well as relations within subsurface communities, may be more easily studied than those amongst surface species and communities (see also Vandel, 1965).

Karst hydrologists have for many years used the flow of groundwater to study inaccessible karst aquifers. The rapidity with which water and water waves flow through the aquifer, as well as the temperature and geochemical characteristics of the water entering and leaving the aquifer, yield information concerning the character of that aquifer (see Brown et al., 1969; Brown 1972; Shuster and White, 1971; among others). The possibility of gleaning similar information from the flow of air has been little studied, due in part to the substantially increased complexity of the mathematical description of the flow. Wigley (Wigley et al., 1966; Wigley, 1967; Wigley and Brown, 1969) has done considerable work on estimating total permeability of aquifers from velocity and pressure variations of air at the entrance to a cave in Australia, while Plummer (1962, 1969) has attempted estimates of cave volume from the resonance of certain caves. Both these studies, however, deal with non-dynamic caves, and work on

dynamic caves is virtually non-existent. Since a knowledge of the microclimatic conditions to be expected in dynamic caves would be vital to the use of air-flow in the remote sensing of karst aquifer characteristics, a dynamic model should be of interest to karst hydrologists.

Much akin to the cave atmosphere is the mine atmosphere, which is of great interest to the mining engineer. To ensure safe and comfortable working conditions, the natural microclimatic conditions found in mines must be ascertained and, if necessary, altered by mechanical ventilation. While mines are found in a much broader spectrum of climatic and geologic conditions, the same basic principles are responsible for the determination of microclimatic conditions in both mines and caves.

## Basic Principles

Having indicated the importance of subterranean microclimatic conditions, one might expect a question as to what determines these conditions. The answer has been intimated by the statement that cave temperatures approximate the mean annual surface temperature; hence the cave atmosphere is not a closed system, but rather is directly connected with and to a large extent determined by the free atmosphere. The nature of this relationship is dependent upon the morphology of the given cave, especially the number of entrances. The classical division of caves

into "static" and "dynamic" was popularised by Geiger (see for instance Geiger, 1966). By implication, static caves (those with only one entrance) do not vary microclimatically. Wigley and Brown (in press) point out that this definition is inadequate, for one-entrance cave systems often have very active air circulations (see Conn, 1966; Wigley et al., 1966; Wigley, 1967; Wigley and Brown, 1969), but for simplicity, this terminology will be maintained. Figure 1 shows an idealised static cave. As external pressure (P[ext]) rises, the internal pressure measured at the surface (P[int]) must also rise to maintain equilibrium. Otherwise a pressure differential could exist at the entrance. In crder for P[int] to increase, air must flow into the cave from the surface. This air circulation is fundamental to the determination of speleomicroclimatic conditions: "Wind is not only the most striking phenomenon of weather in caves, it is also its determining factor..." (Bögli and Franke, 1968, p. 39). Likewise, if P[ext] < P[int], there is a flow of air out the entrance. Thus air circulation and its effect on speleomicroclimatic conditions are, in static caves, related primarily to external barometric pressure changes.

"Dynamic" caves are those with two or more entrances vertically separated by at least a few meters. A markedly different relationship between the cave and free atmosphere exists here, for air need not flow in and out the same entrance. As previously stated, speleomicroclimatic

parameters are considerably less variable than their surface counterparts. This is due in large part to the relative invariability of the rock temperature within the cave. Assume for the moment that the internal and external atmospheres are balanced; that is, they are in static equilibrium (see Figure 2). Now increase the average external temperature. A pressure imbalance is created such that air flows into the upper entrance and out the lower entrance. If a static model is assumed, the pressure difference at either entrance (i.e., the pressure differential that would exist across any barrier within the cave that completely stopped the flow of air) is the difference in weight of a column of cave air and a column of external air whose heights are the height difference between the two entrances (Figure 3). As the warmer air flows into the cave, it is cooled by the rock walls of the cave until it is at virtually the same temperature as the rock. At the same time, the rock walls are warmed by the air, but this process is much slower because of the different heat capacity of the rock. If external conditions did not again vary, then the cave and free atmospheres would, over a period of years, once more achieve static equilibrium. External conditions obviously vary seasonally, however, and over such a short period of time there is very little variation in the rock wall temperature. Air circulation, therefore, is strongly directional with air flowing steadily into the upper entrance and out the lower entrance when

external temperatures are greater than the internal rock-wall temperature, a condition found consistently in summer. In winter, external temperatures are lower than the internal wall temperature and airflow is in the opposite direction. This dynamic wind flow is called the "chimney" (or "stack") effect because of the strong analogy (under winter conditions) to air flow in a chimney. Air within the chimney (cave) is warmer than external air so a pressure imbalance exists which draws air into the fireplace (lower entrance) and pushes it out the top of the chimney (upper entrance).

## Subject of Thesis

This thesis will be concerned specifically with the theoretical prediction of microclimatic parameters in an ideal dynamic cave, given the external atmospheric conditions and the cave morphology. It is recognised that in practice the air circulation in any particular cave may often be a combination of both static and dynamic processes,[2] but the concern here will be for an idealised two-entrance system whose internal atmosphere is conditioned by dynamic influences only. The proposed model will therefore be a dynamic one based on the principles of thermodynamics and heat and mass transfer. This is a significant departure from most previous work, which has assumed static or quasi-static models for the sake of simplicity. Both compressible and incompressible steady flow will be investigated. Since, as previously stated, the

prediction of subterranean atmospheric conditions is of interest to speleologists, biologists, karst hydrologists, and mining engineers alike, the results of this thesis should be of considerable significance to a number of disciplines.

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[2] While barometric pressure changes and the chimney effect are the predominant causes of air circulation, other causes should not be overlooked. Winds induced by the surface drag of flowing water within caves are very common (see Trombe, 1952; Myers, 1962), and Helmholtz resonance has been observed in many caves (Plummer, 1962, 1969; Moore and Nicholas, 1964; see also Schmidt, 1958). For further discussion of the types of winds found in caves, see Trombe (1952), Myers (1962) and Trimmel (1968).

## II - PREVIOUS RESEARCH

### Static Theories

Although the existence of the chimney effect in caves has been known for years, few researchers have attempted to describe it mathematically. Most are content to point out that, in summer, cave air is colder than external air, so it descends; while in winter, the reverse occurs. Nearly everyone who has attempted to model the chimney phenomenon has chosen to treat density (or pressure) differences in terms of hydrostatics. While it is obviously incorrect to use hydrostatics in a dynamic situation, the approach is understandable due to a persistent lack of understanding of the principles involved and due to the mathematical simplicity which this approach affords.

At the turn of the century, the cave atmosphere was thought of as distinct from the free atmosphere: "...static influences, especially those due to differences in temperature, were regarded as decisive..." in determining air circulation (Bögli and Franke, 1968, p. 39). One of the first to use this approach was Simonys (1913). He equated the average kinetic energy of the cave air with the work required to move a parcel of air of unit length through the cave. This model, however, is obviously a gross simplification. In his classical Traité de Spéléologie, Trombe (1952) essentially reiterated the conventional static

model. Rather than actually calculate velocities, he (and most subsequent proponents of the static model) concerned himself with the motive force behind the air movement, deriving the formula

PDIFF = (0.1)(H)(273)((D[int]/T[int])-(D[ext]/T[ext]))

where PDIFF is the pressure difference (in the unfortunate dimensions of gms force/$cm^2$) between internal and external columns of air (the motive force); H is the elevation difference between the two entrances (meters), D[int](gms/liter) and T[int](°K) are the internal air density (corrected to 0°C) and temperature; and D[ext] and T[ext] are the corresponding external parameters. The D's and T's are assumed to be average values taken at an elevation halfway between the entrances, on the presupposition that they vary linearly with elevation. This is definitely an over-simplification in a dynamic situation, and represents a basic flaw in Trombe's approach. Nevertheless, this very simplicity has attracted many individuals to adopt the static model. Montoriol Pous (?), in explaining meteorological observations in a Spanish cave, adopted Trombe's arguments without modification. More recently, however, Cigna (1963, 1965) has modified Trombe's model to include the effects of humidity on air density. Nothing, however, is mentioned about density variations other than those directly attributable to changes in elevation.

Another proponent of the static theory, Eraso (1965), has developed a nomogram for cave climate calculations. Basically, he treats cave and surface air masses as two separate entities which are mixed in the entrance area to produce an air mass differing in temperature and density from both original air masses. The nomogram is intended to simplify calculations of the resulting temperature and density. In effect, as Wigley and Brown (1971) point out, the nomogram states that the transition or "cold zone" temperature (zone of significant temperature fluctuation near the surface) is equal to the external adiabatic wet-bulb temperature, but this is fundamentally incorrect: "...the agreement between the theory of Eraso and observation is [merely] fortuitous..." (Wigley and Brown, 1971, p. 314).

Dynamic Theories

In the mid-1960s, speleologists began to understand that microclimatic parameters such as temperature were not constant within caves, but varied significantly even several thousand feet from the entrance. The previous assumption was that internal temperatures away from the entrance were governed by the transmission of surface conditions through solid bedrock (specifically limestone). Since there is little effect from seasonal fluctuations beyond a depth of fifty feet (see for instance Cropley, 1965),[3] the conclusion

remained that deep cave temperatures are essentially invariable. While agreeing with this basic argument, Cropley (1965) actually observed variations of ±1°F over a mile from the entrance of several caves in West Virginia. He concluded that three temperature zones existed in these caves. Zone 1 is the zone governed by entrance effects. Zone 3 is the zone of constant temperature where conditions are controlled by heat transmitted through limestone from the surface. Zone 2, however, is a dynamic zone where temperatures vary seasonally with an exponentially decreasing amplitude as one progresses into the cave from any entrance which draws air.

Cropley does not directly discuss wind velocity, however, and refrains from a theoretical discussion of temperature distribution. But for the purposes of this thesis, since the chimney effect is dependent upon an internal-external temperature differential, the temperature distribution within a cave is critical to the model to be developed. Wigley and Brown (1971) have succeeded in adequately predicting cave temperatures by including (in a dynamic model) the effects of condensation and evaporation, based on heat and mass transfer theory. They assume a constant wall temperature and show that the difference between the air temperature and the internal wall temperature decays quasi-exponentially to zero as one

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[3] Lange (1954), extends this limit deeper than 50 ft., but the argument is still the same.

proceeds along the cave passage in the direction of the wind. The rate of decay was found to be dependent upon passage size and wind velocity. This study by Wigley and Brown will serve as the basis for the present thesis.

## III - RELATED RESEARCH

A problem similar to the determination of microclimatic parameters of the cave atmosphere is found in the planning of mine ventilation. Mines with two or more connections to the surface are analogous to dynamic caves. To maintain healthy working conditions, many mines must be artificially ventilated, and the degree to which the natural ventilation (due to the chimney effect) helps or hinders this artificial ventilation is of considerable importance.

Mine engineers have been concerned with mine ventilation for a much longer period than have microclimatologists with caves, and it is only to be expected that their analysis of the chimney effect (and its effect upon the mine atmosphere) is somewhat more refined than the microclimatologists'. The major advance of engineers in this area has been the calculation of what is termed the "natural ventilation pressure." In the absence of artificially-induced density differences, this may be defined as the pressure difference (due to density differences) between two columns of air of equal height, (technically the height of the atmosphere above the lower entrance, but generally taken as the vertical separation of the two entrances) one within and the other exterior to the mine. This is the shaft-and-adit case, which best approximates a dynamic cave. In mine ventilation literature, however, the natural ventilation pressure (NVP) also refers to the pressure difference

resulting from density differences induced by artificial ventilation (as opposed to the pressure differential due to the fan itself). Because the two entrances to the mine need not be at different elevations (see Figure 4), this situation is markedly different from dynamic caves (where no artificial ventilation exists).

Early attempts to calculate natural ventilation pressures differed little from static models of the chimney effect in caves. Beard (1920) employed the formula

$$P = (1.3273)(B)(H)((1/R[ext]) - (1/R[int]))$$

where P is the pressure difference (lbs/ft$^2$), B is the atmospheric pressure (in. of Hg), H the height of the columns (ft), and R[ext] and R[int] the average temperature (°R) of the external and internal columns. This is equivalent to the static model of Trombe (see page 8), with the exception that temperature rather than density is averaged over height. Penman and Penman (1947) adopt the same approach although their pressure differential is couched in terms of a motive column. This motive column may be visualised by assuming that the two static columns have the same density, but unequal heights. The pressure differential between the two columns, then, is merely this common density times the difference in height of the two columns, times the acceleration due to gravity.

The natural ventilation pressure refers only to the

static pressure difference between two columns of air. Should air circulation be allowed between the two columns (that is, between the mine or cave and the surface), the ensuing air velocity may be related to the natural ventilation pressure by the following formula, which is ascribed to Atkinson (1886, as cited in Weeks, 1926):

$$(P)(A) = (K)(S)(V)^2 ,$$

where

$P$ = pressure difference in lb/ft$^2$

$A$ = cross-sectional area of passage in ft$^2$

$S$ = rubbing surface in ft$^2$

$V$ = air velocity in ft/min

$K$ = coefficient of friction .

The value of K is dependent upon the units used and includes the average density of the air. The equivalent formula using metric units (i.e., the appropriate friction coefficients) may be found in Stǒces (1954).

This static approach to air flow in mines is prevalent even today. McElroy (1950) and Harris and Kingerly (1973) make minor allowances for flow-induced temperature changes, but their models are by no means dynamic. McElroy acknowledges the effects of moisture on air density, but considers its inclusion into his model unwarranted. Harris and Kingerly give the following formula for the density of moist air:

$$\text{Density} = (1.327)(1/T)(B-0.378f)$$

where the density is in $lb/ft^3$; T is the temperature in degrees Rankine; B the atmospheric pressure in inches of mercury; and f the vapor pressure (in inches of mercury) at the dewpoint temperature. They then state that temperature and pressure are too variable for this sort of accuracy, so that the vapcr pressure can be ignored. Their model then becomes identical to that of Beard (1920), as previously mentioned.

Not all mine engineers follow this school of thinking, however. The departure from static modelling began with F.B. Hinsley (1938, as cited in Williams, 1960; 1943a,b,c,d). Hinsley recognised natural ventilation as a problem involving the thermodynamics of compressible fluid flow. Heat added to (taken frcm) the mine air is balanced by the work done by (on) the air and by the change in internal energy of the air. The work done includes changes in kinetic and potential energies, changes in pressure energy (Hinsley's terminology), and the work done against friction. The heat added includes the net external heat added, the frictional heat, and any mechanical energy added (all variables are measured per unit weight). Thus the governing energy equation, between any two cross-sections i and j, is:

$$\begin{aligned} H(1,2) + M(1,2) + F(1,2) &= [Z(2)-Z(1)] + [\{U(2)^2 - U(1)^2\}/2G] \\ &\quad + [P(2)V(2) - P(1)V(1)] + \underline{F}(1,2) \end{aligned}$$

$$+ [E(2)-E(1)]$$

where

| | | |
|---|---|---|
| H(i,j) | = | net heat added between i and j |
| M(i,j) | = | mechanical energy added between i and j |
| F(i,j) | = | frictional heat added between i and j |
| <u>F</u>(i,j) | = | work done against friction between i and j |
| Z(i) | = | potential energy at i |
| U(i) | = | air velocity at i |
| P(i) | = | pressure at i |
| V(i) | = | specific volume at i |
| E(i) | = | internal energy at i |
| G | = | acceleration of gravity . |

(Care should be taken to assure compatibility of dimensions when mixing heat and mechanical energy terms.) In the case of natural ventilation, there is no mechanical energy involved (M(1,2) = 0) and the work done against friction is assumed to be exactly the heat gained from friction (that is, there is no loss of heat to external sinks; <u>F</u>(1,2) = F(1,2)). If

W(i,j) = work done on or by the fluid between i and j ,

the preceding equation, ignoring variations in potential and kinetic energy reduces to

$$H(1,2) + F(1,2) = W(1,2) + [E(2)-E(1)].$$

Hinsley showed that the work done was

$$W(1,2) = \int_1^2 PdV ,$$

so it is necessary to know the variation of pressure with specific volume to be able to calculate this work.

If an ideal mine is assumed (as pictured in Figure 5), then air moving through the mine (for whatever reason) goes through the following thermodynamic cycle:

1) Air enters the mine and descends, gaining heat at the expense of potential energy.

2) At the bottom of the downcast shaft, air passes through the workings and gains heat from the strata.

3) The air rises up the upcast shaft, losing internal energy to potential energy.

4) The air at the top of the upcast shaft is returned to the state of the air at the top of the downcast shaft.

On a pressure-specific volume (P-V) graph, this cycle is ideally that of Figure 5. The net work done per lb of air (the area enclosed by the cycle), times the average value of the air density, yields the natural ventilation pressure.

Several other authors have adopted the thermodynamic approach; among them, Williams (1960), and Hall (1953, 1967). While the approach is basically correct, there are numerous problems associated with the actual calculations. If actual field measurements are taken and plotted on a P-V diagram, the determination of the natural ventilation pressure is reasonably straightforward. To calculate the NVP from surface conditions alone, without actually measuring

the pressure-volume variation, is considerably more difficult. Hinsley (1943a,b,c,d) assumed that flow in the shafts was frictional adiabatic (Hinsley's terminology - adiabatic modified by frictional effects), and that the collars of the upcast and downcast shafts were at the same elevation so potential energy differences would sum to zero. Hall (1967) looks extensively at these assumptions and attempts to analyse the effects of removing them. Hall shows that the ideal mine cycle, with no friction, with collars at the same elevation, with no moisture, and with no net loss of kinetic energy through the cycle, is actually the Joule or Brayton cycle. This cycle, which consists of two reversible adiabatic processes joined by two isobaric processes, is pictured in Figure 5 on a P-V diagram. If these factors are taken into consideration, the cycle should look like Figure 6, but Hall is unable to evaluate their simultaneous effects and must resort to a step-by-step evaluation resulting in a cycle like that of Figure 7. Assumptions are also made in Hinsley's model concerning the character of the heat transferral in both the shafts and the workings, and Hinsley says very little concerning mine entrances at different elevations, the situation most analogous to dynamic caves. Clearly improvements can be made to both Hinsley's and Hall's models.

## IV - THE COMPRESSIBLE MODEL

### The Approach

Much of the difficulty with mine ventilation studies involves the necessity for calculating the natural ventilation pressure despite the inability to integrate all the involved factors into a continuous process. While many speleomicroclimatologists have followed the approach of calculating the NVP and <u>then</u> the velocity of the air flow from the Atkinson Formula, this is circular reasoning, because the velocity of the air determines largely the pressure variation that exists (due primarily to the variation of temperature with velocity as will be seen later). A more valid approach would aim at calculating velocities first. For the speleologist, this would be enough; the cave atmosphere is determined. For the mine engineer, once the mine portion of the P-V cycle is known, calculation of the surface parameters completes the cycle and the NVP may then be calculated.

The major obstacle to integrating all the factors involved in the calculation of air velocities is the problem of estimating the net heat exchange between the air and the rock strata. However, as previously pointed out, this has been solved theoretically by Wigley and Brown (1971). It is therefore possible to determine the parameters of the cave atmosphere under steady-state conditions, given the external

atmospheric conditions at the two entrances and certain non-dynamic properties of the particular cave. The approach is via the thermodynamics of compressible fluid flow (or gas dynamics), and the basic model is described in the next section.

The Model

To apply the techniques of gas dynamics to an actual cave would dc little to elucidate the principles involved. Variations in passage gradient and cross-sectional area, and the effects of passage bending would serve only to mask the actual chimney process. To avoid such difficulties, the passage will be assumed to have a constant radius and gradient; but in order to give the reader some insight as to their influence, these assumptions will not be immediately introduced. The effects of bends and of momentum losses due to rapid changes in passage size, direction, or configuration will at no time be considered. For information in this area, the reader is referred to Shapiro (1953), from whom much of the following is taken. Radial variations in the parameters are of no importance here, so the flow will be treated as one-dimensional with all parameters being averages (for a more precise definition of average, see Wigley and Brown, 1971) over each cross-section. For the present, only the steady-state case will be investigated.

The effects of moisture in the air deserve special mention. In most situations, moisture will be added to (or

taken away) from the air by condensation (or evaporation). This creates a substantial heat source (or sink) which cannot be neglected. Density differences due to moisture, however, are minor and will be ignored, so that the mass rate of flow is constant.

The Dynamics and Thermodynamics

The ideal dynamic cave, with constant gradient and cross-section, is pictured in Figure 8, and all relevant parameters are explained in Table 1. The basic procedure is to look at an incremental section of passage (called the control volume - see Figure 9) of length dx, and describe the change in density, pressure, and air velocity in terms of the temperature and the friction along the walls. The solution is obtained from the simultaneous solution of the equations of state, continuity, and momentum. For simplicity, the concepts of Mach number and stagnation state are introduced; and, for completeness, the energy equation, although not required for the solution, is also derived.

For a perfect gas, which _dry_ air approximates (corrections for moist air, to avoid a variable gas constant, will be made later), the equation of state is

$$P = D(Rg)T \quad , \tag{4.1}$$

from which logarithmic differentiation yields

$$\frac{dP}{P} = \frac{dD}{D} + \frac{dT}{T} \quad . \tag{4.2}$$

The Mach number is defined to be

$$M = V\left[k\,(Rg)\,T\right]^{-0.5} \quad . \tag{4.3}$$

Thus by logarithmic differentiation,

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} - \frac{dT}{T} \quad . \tag{4.4}$$

since k is constant for a perfect gas. The mass rate of flow is

$$w = DAV \quad , \tag{4.5}$$

so again by logarithmic differentiation,

$$0 = \frac{dD}{D} + \frac{dA}{A} + \frac{dV}{V} \quad . \tag{4.6}$$

since w is constant.

The momentum equation is somewhat more complicated. "The net force acting on the material within the control surface is equal to the increase of momentum flux of the streams flowing through the control surface" (Shapiro, 1953, p. 224). Thus, assuming the divergence of the walls is small (angle of divergence approximated as 0), we have

$$PA + PdA - (P + dP)(A + dA) - t[wall]dA[wall] - dX$$
$$= w(V + dV) - w(V) \quad . \tag{4.7}$$

The wall friction factor is defined to be

$$f = \frac{2t[wall]}{DV^2} , \tag{4.8}$$

so

$$t[wall] = \frac{fDV^2}{2} . \tag{4.9}$$

Further, by defining B as the mean hydraulic diameter (the cross-sectional area of the passage divided by the perimeter of the cross-section), we have

$$\frac{dA[wall]}{A} = \frac{4dx}{B} . \tag{4.10}$$

Thus, from (4.7), (4.9), and (4.10), and neglecting second order terms,

$$wdV = -AdP - \frac{2ADV^2fdx}{B} - dX . \tag{4.11}$$

But, from (4.1) and (4.3),

$$DV^2 = kPM^2 , \tag{4.12}$$

so from (4.5), (4.11) and (4.12), we get

$$kPAM^2\frac{dV}{V} = -AdP - \frac{2kPAM^2fdx}{B} - dX . \tag{4.13}$$

Dividing by PA and rearranging,

$$\frac{dP}{P} = -kM^2\frac{dV}{V} - kM^2\frac{2fdx}{B} - \frac{dX}{PA} . \tag{4.14}$$

But $X$ is the component of weight working against the flow, so if $Z[x]$ is the height above the lower entrance at $x$ distance intc the cave,

$$dX = DAGZ'[x]dx \quad , \tag{4.15}$$

and thus, frcm (4.14), (4.15), and (4.1),

$$\begin{aligned} \frac{dP}{P} &= -kM^2\frac{dV}{V} - kM^2\frac{2fdx}{B} - \frac{DGZ'[x]dx}{P} \\ &= -kM^2\frac{dV}{V} - kM^2\frac{2fdx}{B} - \frac{GZ'[x]dx}{(Rg)T} \end{aligned} \tag{4.16}$$

The energy equation, while nct required for solution of the problem, is nevertheless instructive. For flow through the contrcl surface, the rate of heat flow is

$$\begin{aligned} wdQ &= w(h + dh) - wh + \left[\frac{V^2}{2} + \frac{dV^2}{2}\right]w \\ &\quad - \frac{wV^2}{2} - wc[p]L(z + dz) \\ &\quad + wc[p]Lz \\ &= wdh + \frac{wdV^2}{2} - wc[p]Ldz \quad , \end{aligned} \tag{4.17}$$

where $dQ$ is the net heat per unit mass added to flow by conduction cr radiation from external sources. Dividing by $w$, we get

$$dQ = dh + \frac{dV^2}{2} - c[p]Ldz \quad . \tag{4.18}$$

We may break $dh$ down to

$$dh = -dh[pr] + c[p]dT \quad , \tag{4.19}$$

yielding, upon rearrangement,

$$dQ + dh[pr] + c[p]Ldz = c[p]dT + \frac{dV^2}{2} \quad . \tag{4.20}$$

Division by c[p]T gives

$$\frac{dQ + dh[pr] + c[p]Ldz}{c[p]T} = \frac{dT}{T} + \frac{dV^2}{2c[p]T} \quad , \tag{4.21}$$

and substitution from (4.4) and the relationship between Rg, c[p], and k yields

$$\frac{dQ + dh[pr] + c[p]Ldz}{c[p]T} = \frac{dT}{T} + \frac{(k-1)M^2}{2}\frac{dV^2}{V^2} \quad . \tag{4.22}$$

Thus, the heat input from the strata plus the heat input (or output) from condensation (evaporation), plus the energy lost to the gain in elevation (negative), is described totally by the temperature and velocity, as would be be expected.

It is advantageous here to introduce the concept of stagnation state, which is defined as that state arrived at by isentropically reducing the flow to zero velocity. From (4.18), neglecting potential energy and phase changes,

$$ho = h + \frac{V^2}{2} \quad , \tag{4.23}$$

from which the following relationship can be derived:

$$T_o = T + \frac{V^2}{2c[p]} \quad . \tag{4.24}$$

From (4.3), this may be written

$$T_o = T\left[ 1 + \frac{(k-1)M^2}{2} \right] \quad . \tag{4.25}$$

Similarly,

$$P_o = P\left[ 1 + \frac{(k-1)M^2}{2} \right]^{k/(k-1)} \quad , \tag{4.26}$$

$$D_o = D\left[ 1 + \frac{(k-1)M^2}{2} \right]^{1/(k-1)} \quad . \tag{4.27}$$

Rather than solve directly for density, pressure, and velocity, it is easier to solve for the Mach number variation alcng the passage (to be discussed later). Then, from equations (4.2), (4.3), (4.4), and (4.6) and the variation of the temperature along the passage, the density, pressure, and velocity variations and their related stagnation properties may be obtained.

The solution for density from (4.4) and (4.6), assuming the Mach number and temperature variation to be known, is

$$\begin{aligned} \frac{dD}{D} &= -\frac{dA}{A} - \frac{dV}{V} \\ &= -\frac{dA}{A} - \frac{dV^2}{2V^2} \end{aligned}$$

$$= -\frac{dA}{A} - \frac{dM^2}{2M^2} - \frac{dT}{2T} . \tag{4.28}$$

If (4.28) is integrated between any two sections 1 and 2 and exponentials taken, we arrive at

$$\frac{D(2)}{D(1)} = \frac{A(1)}{A(2)} \frac{M(1)}{M(2)} \left[ \frac{T(1)}{T(2)} \right]^{0.5} . \tag{4.29}$$

Similarly, for pressure, from (4.2) we have

$$\frac{P(2)}{P(1)} = \frac{D(2)}{D(1)} \frac{T(2)}{T(1)} . \tag{4.30}$$

But substitution from (4.29) gives

$$\frac{P(2)}{P(1)} = \frac{A(1)}{A(2)} \frac{M(1)}{M(2)} \left[ \frac{T(2)}{T(1)} \right]^{0.5} . \tag{4.31}$$

The solution for Mach number variation is somewhat more complicated. Starting with equations (4.4), (4.6), and (4.2), we get

$$\begin{aligned} \frac{dM^2}{M^2} &= \frac{dV^2}{V^2} - \frac{dT}{T} \\ &= -2\frac{dD}{D} - 2\frac{dA}{A} - \frac{dT}{T} \\ &= -2\frac{dP}{P} - 2\frac{dA}{A} + \frac{dT}{T} . \end{aligned}$$

Substitution from the momentum equation, (4.16), gives

$$\frac{dM^2}{M^2} = kM^2\frac{dV^2}{V^2} + kM^2\frac{4fdx}{B} + \frac{2GZ'[x]dx}{(Rg)T} - \frac{2dA}{A} + \frac{dT}{T} .$$

and from (4.4),

$$\frac{dM^2}{M^2} = kM^2\frac{dM^2}{M^2} + (1 + kM^2)\frac{dT}{T} + kM^2\frac{4fdx}{B}$$

$$+ \frac{2GZ'[x]dx}{(Rg)T} - 2\frac{dA}{A} \quad .$$

By collecting $\frac{dM^2}{M^2}$ , this yields

$$(1 - kM^2)\frac{dM^2}{M^2}$$

$$= (1 + kM^2)\frac{dT}{T} + kM^2\frac{4fdx}{B} + \frac{2GZ'[x]dx}{(Rg)T} - 2\frac{dA}{A} \quad .$$

$$\frac{dM^2}{M^2} = \left[\frac{1 + kM^2}{1 - kM^2}\right]\frac{dT}{T} + \left[\frac{4kM^2}{1 - kM^2}\right]\frac{fdx}{B}$$

$$+ \left[\frac{2}{1 - kM^2}\right]\frac{GZ'[x]dx}{(Rg)T}$$

$$- \left[\frac{2}{1 - kM^2}\right]\frac{dA}{A} \quad . \tag{4.32}$$

Only under very specific conditions may this equation be solved analytically. For a numerical solution, we first invoke the simplifications previously mentioned. The cross-sectional area becomes constant with constant radius R (necessitated by the temperature solution discussed later). Thus $dA/A = 0$ and $B = 2R$. Further, the gradient is constant, so $Z[x] = x\sin\theta$ and $Z'[x] = \sin\theta$. With these simplifications, equation (4.32) becomes

$$\frac{dM^2}{M^2} = \left[\frac{1 + kM^2}{1 - kM^2}\right]\frac{dT}{T} + \left[\frac{2kM^2}{1 - kM^2}\right]\frac{fdx}{R}$$

$$+ \left[\frac{2}{1 - kM^2}\right]\frac{G\sin\theta dx}{(Rg)T} \quad . \tag{4.33}$$

For manipulative simplicity, let

$$FI = \frac{M^2(1 + kM^2)}{1 - kM^2} \quad , \tag{4.34}$$

$$FII = \frac{2kM^4}{1 - kM^2} \quad , \tag{4.35}$$

$$FIII = \frac{2M^2}{1 - kM^2} \quad . \tag{4.36}$$

Then (4.33) becomes

$$dM^2 = FI\frac{dT}{T} + FII\frac{fdx}{R} + FIII\frac{G\sin\theta dx}{(Rg)T} \quad . \tag{4.37}$$

By assuming that the coefficients of dT and dx are constant at their mean value over the domain of integration (trapezoidal rule), equation (4.37) may be integrated between sections 1 and 2 to yield

$$M(2)^2 - M(1)^2$$

$$= \left[\frac{FI(2)}{T(2)} + \frac{FI(1)}{T(1)}\right]\left[\frac{T(2) - T(1)}{2}\right]$$

$$+ \left[ \frac{f[FII(2) + FII(1)]}{R} \right] \left[ \frac{x(2) - x(1)}{2} \right]$$

$$+ \left[ \frac{FIII(2)}{T(2)} + \frac{FIII(1)}{T(1)} \right] \frac{G\sin\theta}{(Rg)}$$

$$\bullet \left[ \frac{x(2) - x(1)}{2} \right] . \qquad (4.38)$$

Now from equations (4.25), (4.26), (4.27), (4.30), (4.31), and (4.38), a complete solution for density, velocity, pressure, and the related stagnation properties is available for any cross-section (2) in terms of some previous cross-section (1) and the temperature distribution. As FI, FII, and FIII all contain M(2), the solution for M(2) in equation (4.38) will of necessity be an iterative one.

## Temperature Variation - Dry Air

For illustrative purposes, let us initially look at the case where there is no moisture whatsoever in the cave or free atmosphere. In this instance, the variation of temperature, assuming fully-developed turbulent flow, is expressed (in the case of a horizontal passage) by the differential equation

$$\frac{dT}{dx} = \frac{(TWALL - T)}{xo} \qquad (4.39)$$

(see Shapiro, 1953; Wigley and Brown, 1971). Wigley and

Brown (1971) have shown that, for smooth-walled passages,

$$xo = \frac{F(Pr)(Re)}{2(Nu)} , \tag{4.40}$$

where Pr is the Prandtl number, Re the Reynolds number, and Nu the Nusselt number of the flow. Empirical relationships have been worked out (see for instance Kays, 1966) which yield the Nusselt number in terms of the Prandtl and Reynolds numbers, but the correct relationship depends upon the nature of the heat flow between rock wall and air. It is necessary, therefore, to look at what determines the temperature of the rock wall.

To analyse TWALL, consider what conditions would exist within the solid rock mass in the absence of a cave. "The thermal condition of a rock mass at a given time is a function of its conductivity, density, heat capacity, rate of internal heat generation and loss, temperature fluctuations of the surface, and the regional geothermal gradient" (Lange, 1954, p.21). By assuming a level-surfaced homogeneous rock mass of constant thermal conductivity and diffusivity, Lange has calculated the variation of temperature with depth from a sinusoidal variation in surface temperature. If k is the thermal conductivity of the rock, and d and c[p] its density and heat capacity (constant pressure), then the temperature $\theta$ at depth z and time t resulting from a surface temperature fluctuation of Ncoswt is

$$\theta(z,t) = N\exp(-mz)\cos(wt - mz) \,, \tag{4.41}$$

where

$$m = \left[ \frac{wdc[p]}{2k} \right]^{0.5} . \tag{4.42}$$

This fluctuation is superimposed upon a mean geothermal gradient which is assumed to be linear with depth. The period of the surface fluctuation is $2\pi/w$ and the amplitude is N, and the thermal diffusivity $k/dc[p]$ depends upon the particular rock. For certain marbles that Lange investigated, the thermal diffusivity was 0.00502 $m^2/hr$, so using this value, a diurnal temperature variation of 20°C would yield a subsurface variation of

$$\theta(z,t) = 10\exp(-5.1z)\cos(.262t - 5.1z) \,. \tag{4.43}$$

From equation (4.43), by setting $(.262t - 5.1z) = 0$, the depth at which a 2°C variation exists (as a result of the 20°C surface variation) is 0.45 meters. Clearly, daily surface variations will have no significant effect upon rock temperatures at depth.

Now let us assume a 40°C annual variation. Equation (4.41) now becomes

$$\theta(z,t) = 20\exp(-0.267z)\cos(.000717t - 0.267z) \,, \tag{4.44}$$

so the depth at which a 2°C variation is felt is 11.2

meters. Again, seasonal effects are minimal with depth. This process may be continued indefinitely, and it becomes reasonably obvious that only cycles of very long duration can possibly affect rock temperatures to any depth. Thus, for our purposes, the geothermal gradient may be considered time-independent.

Despite the lack of dependence on temporal variations, the determination of rock temperature is still complex. In low-lying karst areas, such as central Kentucky, the problem is not too difficult; the temperature at depth may be extrapolated from the average surface temperature and the regional geothermal gradient, the latter usually being between 0.01 and 0.05°C/meter (see Carslaw and Jaeger, 1959; as cited in Ford et al., 1975). In alpine areas, however, the change of mean annual temperature with elevation, and especially the effects of a highly irregular surface, severely complicate determination of the rock temperature at depth, and such an endeavor would be beyond the scope of this thesis. One approximation would be to assume the surface was level and extrapolate from the local mean annual temperature and the regional geothermal gradient (see Figure 10). The error should not be large if the local surface is not steep and the relief not great. This approach is common in mine engineering, where surface irregularities are often small in comparison to mine depths. Caves, however, are considerably more restricted in their location; that is, they are formed by groundwater flow and hence do not extend

significantly deeper than the lowest point at which the limestone outcrops (at the time of their formation), which is the approximate point at which their waters resurface. Thus, cave depths are generally of the same order of magnitude as (or less than) the local topographic relief, and the variation of mean annual surface temperature with elevation significantly affects the cave rock temperature at depth. A possible alternative approximation would be to assume that the temperature of the rock varies linearly with the distance <u>normal</u> to the mean surface (see Figure 10). Many caves descend or ascend at approximately the same gradient as the surface; hence estimates of the rock wall temperature by this method would yield reasonable results.

The simplest approximation is to assume an isothermal rock mass. For shallower caves in low-lying areas, this is a reasonable approximation. For caves in alpine areas or at great <u>depth</u> in low-lying regions, however, the isothermal case can only be considered a first approximation. Those interested in further approximation methods are referred to Kays (1966), where an in-depth discussion of the relationship between the Nusselt, Prandtl, and Reynolds numbers (for flow through constant diameter pipes) is found. For the purposes of this paper, only the isothermal case will be investigated, because of the mathematical simplicity it affords.

The foregoing discussion concerned rock temperatures at

depth in a homogeneous rock mass. The introduction of a dynamic cave into this rock mass necessarily alters somewhat the temperature distribution within the rock, for not only does the rock-wall temperature affect the internal air temperature, but also the reverse is true. The effects are most noticeable in the entrance area, where the internal air temperature differs most from the temperature of the rock mass. Beyond the entrance zone, however, the effects, should be minimal because the temperature difference rapidly becomes quite small.

For the sake of mathematical tractability, Wigley and Brown (1971) ignore heat transfer within the wall and replace it with the boundary condition that the wall temperature TWALL is constant. Under these somewhat stringent conditions, the relationship between the Nusselt, Prandtl, and Reynolds numbers (for a constant pipe diameter) is

$$Nu = (0.021)(Pr)^{0.6}(Re)^{0.8} \tag{4.45}$$

(see Kays, 1966; as cited in Wigley and Brown, 1971). This, with equation (4.40), yields

$$xo = (24)(Pr)^{0.4}(Re)^{0.2} R . \tag{4.46}$$

From the definitions of Pr and Re, this in turn gives

$$xo = (28)\left[\frac{(\text{coeff of viscosity})\,c[p]^2}{(\text{thermal conductivity})^2}\right]^{0.2} \cdot (R)^{1.2}\,(DV)^{0.2} \quad . \qquad (4.47)$$

While D and V vary along the passage, DV is constant and equal to w/A. Further, for dry air,

$$c[p] = 1004\ \text{m}^2/\text{sec}^2\text{-}^{o}\text{C} \qquad \text{(Hess, 1959)},$$

and since the coefficient of viscosity and the thermal conductivity vary little with temperature and density, a reasonable approximation is gained by considering them constant. At 10°C, their values (by linear interpolation from Weast, 1970) are:

$$\text{coefficient of viscosity} = 1.774(10)^{-5}\ \text{kg/m-sec} \quad ,$$

$$\text{thermal conductivity} = 2.482(10)^{-2}\ \text{J/m-sec-}^{o}\text{C} \quad .$$

Thus, equation (4.47), considering that $A = \pi R^2$, becomes

$$xo = (170)\,(R)^{0.8}\,(w)^{0.2} \quad . \qquad (4.48)$$

The previous formula assumes the passage is smooth-walled. If this is not the case, xo should be corrected with use of the actual friction factor (see Kays, 1966), as follows:

$$xo \quad = \left[ \frac{f[smooth]}{f} \right]^{0.5} (169.86)\,(R)^{0.8}\,(w)^{0.2} \quad . \qquad (4.49)$$

Since TWALL is constant for the isothermal rock mass considered in this paper, and xo is constant for steady flow conditions, (assuming a constant diameter pipe) equation (4.39) may now be solved. The solution (given T=TEXTL at x = 0) is:

$$T \quad = \quad TWALL + (TEXTL - TWALL)\exp(-x/xo) \quad , \qquad (4.50)$$

which describes a simple exponential decay of the entrance air temperature to the rock-wall temperature, with distance into the cave.

To alter this formula for our non-horizontal passage, as Wigley and Brown (1971) state, the appropriate adiabatic lapse rate (in this case, the dry rate) need only be superimposed as a non-dynamic effect. This cannot be the case, however, for any change in temperature due to a change in elevation must affect subsequent temperatures in the cave (by equation (4.39)). The proper method of taking these factors into account is to replace the actual temperature with the potential temperature (Tp), which is the temperature after the process is adiabatically reduced to isobaric conditions. The actual and potential temperatures are related as follows

$$Tp = T\left[\frac{1000}{P}\right]^{R/c[p]} .$$

It may be shown (from the second law of thermodynamics) that equation (4.39) becomes

$$\frac{TdlnTp}{dx} = \frac{(TWALL - T)}{xo} .$$

so that

$$\frac{dT}{dx} - \frac{RTdlnP}{c[p]dx} = \frac{(TWALL - T)}{xo} .$$

But

$$\frac{-RTdlnP}{c[p]dx} = \frac{-RTdP}{Pc[p]dx}$$

$$= \frac{-GdP}{DGc[p]dx}$$

$$= \frac{Lsin\theta dP}{DGdz} .$$

By approximating the density variation with height as that under static loads, this yields

$$\frac{-RTdlnP}{c[p]dx} = Lsin\theta ,$$

so we see that dT/dx in the level (isobaric) case need only be replaced by (dT/dx) + Lsinθ in the non-level case. Solving equation (4.39) with this modification yields

$$T = TWALL - Lxosin\theta + (TEXTL - TWALL - Lxosin\theta)exp(-x/xo) . \qquad (4.51)$$

Temperature Variation - Moist Air

The introduction of moisture to the air complicates substantially the determination of temperature. While dry air and water vapor separately approximate perfect gases, the combination does not (without resort to a variable gas constant dependent upon the humidity of the air). This may be overcome by introducing the concept of virtual temperature, which is defined as "...the temperature at which dry air would have to be in order to have the same density as a sample of moist air, assuming both have the same pressure" (Hess, 1959, p.60). The use of virtual temperatures allows the employment of the equation of state for dry air under all circumstances. The general formula for virtual temperature is:

$$Tv = T(1 + 0.609q) \quad .$$

Another important effect related to the introduction of moisture to the air is the heat addition (or loss) from condensation (or evaporation). Wigley and Brown (1971) have also taken this into consideration and have derived a modified form of the differential equation (4.39) which accounts for condensation and evaporation. This modified equation is:

$$\frac{dT}{dx} = \frac{1}{xo}\left[ TWALL - T - \frac{HE(QWALL - QSPEC)}{c[p]} \right] , \tag{4.53}$$

where

$$\frac{dQSPEC}{dx} = \frac{E(QWALL - QSPEC)}{xo} , \tag{4.54}$$

and E is the fraction of the passage wall that is moist. Generally, E would vary along the passage; hence it would be a function of x. For most applications, however, it is sufficient to assume that E = 1 . Since (4.53) becomes (4.39) when E = 0 , we will retain E in the formula as a constant, so that moist and dry solutions may be compared. The solution to (4.54) under the same assumptions as for dry air (xo and TWALL constant, hence QWALL also constant), is

$$QSPEC = QWALL + (QEXT - QWALL)\exp(-Ex/xo) . \tag{4.55}$$

Substituting this into (4.53) gives

$$\frac{dT}{dx} = \frac{1}{xo}\left[ TWALL - T + \frac{HE(QEXT - QWALL)\exp(-Ex/xo)}{c[p]} \right] , \tag{4.56}$$

which may be solved to yield

$$T = TWALL + (TEXTL - TWALL)\exp(-x/xo) + \frac{HE(QEXT - QWALL)x\exp(-Ex/xo)}{c[p]xo} , \tag{4.57}$$

Again, for our non-horizontal passage, this becomes

$$T = TWALL - Lxo\sin\theta + (TEXTL - TWALL + Lxo\sin\theta)\exp(-x/xo) + \frac{HE(QEXT - QWALL)x\exp(-Ex/xo)}{c[p]xo}, \quad (4.58)$$

Equation (4.53) does not always apply, however. In certain circumstances supersaturation would be predicted by this equation, and alterations must be made to avoid this. If xsat is the point at which saturation is achieved, and TSAT the temperature at that point, then the solution (see Wigley and Brown, 1971, for details) is

$$(1 + [J1])\ln\left[\frac{TSAT - TWALL}{T - TWALL}\right] + [J1][J2](TSAT - T) = \frac{x - xsat}{xo}, \quad (4.59)$$

which, rearranged, becomes

$$T = TWALL + (TSAT - TWALL) \cdot \exp\left[\frac{-x + xsat + xo[J1][J2](TSAT - T)}{(1 + [J1])xo}\right], \quad (4.60)$$

where

$$[J1] = \frac{(QWALL)(H)}{(Rv)c[p](TWALL)^2},$$

and

$$[J2] = \frac{H}{(Rv)(TWALL)^2} \quad .$$

Again, equation (4.60) can be modified for the non-horizontal case to yield

$$T = TWALL - \frac{ac}{bc+1} + \left[ TSAT - TWALL + \frac{ac}{bc+1} \right]$$

$$\cdot \exp\left[ \frac{(bc+1)^2(xsat - x) - (T - TSAT)(xo)b(bc+1)}{a(xo)} \right] , \qquad (4.61)$$

where

$$a = 1 + [J1]$$

$$b = [J1][J2]$$

$$c = I(xo)\sin\theta \quad .$$

Thus equation (4.58) is used when x<xsat and equation (4.61) when x≥xsat. Both equations must be converted to virtual temperature by use of equations (4.52) and (4.55) if they are to be used to calculate the variation in Mach number in equation (4.38). For x<xsat the formula is

$$Tv = T(1 + 0.609(QWALL + (QEXT - QWALL)\exp(-Ex/xo))) , \qquad (4.62)$$

where T comes from equation (4.58). For x≥xsat,

$$Tv = T(1 + 0.609(QSAT[T])) , \qquad (4.63)$$

where T is found by iteration from equation (4.61).

The description of the temperature distribution is now complete. Equations (4.58), (4.61), (4.62), and (4.63) may be used in conjunction with equation (4.38) to describe the variation of Mach number along the passage given suitable boundary conditions to be discussed later. The mass rate of flow is required for the temperature solution, and while the stagnation density will be calculable from the boundary conditions, the velocity (and hence the actual density) must be found iteratively. The friction factor, required for both the description of the temperature variation and the Mach number variation, is discussed below.

## The Friction Factor

The friction coefficient f is the Fanning friction factor, which relates the wall shear stress to the difference between the actual pressure and the stagnation pressure (commonly called the velocity pressure - see equation (4.8)). It is a dimensionless parameter which may be shown by dimensional analysis to depend upon the Mach number, the Reynold's number, and the relative roughness of the passage wall (Thompson, 1972). The dependence of f on the Mach number, however, is generally small, especially when the Mach number is extremely low (<0.1), as is the case here. Thus, f is here considered to be a function only of the Reynolds number and the relative roughness (see Schlichting, 1968; as cited in Chapman and Walker, 1971).

For the steady flow case (with constant pipe radius) being investigated, the Reynolds number is treated as constant, so that f depends only upon the relative roughness. The relative roughness, which is the average height of the wall irregularities divided by the passage diameter, may be expected to vary along the passage. Because the surface irregularities are only average values over a distance, a common simplifying assumption is that the friction factor is constant for a given problem such as this one. Therefore, one need only look up the appropriate value of the friction factor for the type and roughness of the rock. These values are commonly found in any modern mine engineering handbook.

In hydraulics, the most common source for tables of friction factors is Moody (1946). These values, however, are for the Darcy friction factor, rather than the Fanning friction factor. The Darcy friction factor is exactly four times as large as the Fanning, and the reader is cautioned to determine which a given set of tables refers to, as it often is not stated.

For application to caves and mines, however, common engineering sources for friction factors (such as the previously mentioned Moody (1946)) are generally not sufficient, because the range of relative roughness is too low. We turn, therefore, to the mining literature, and should immediately caution the reader as to what will be found. A multitude of definitions exist here for friction

factors. They are <u>all</u> denoted by K and seldcm can one readily determine the relationship between one K and another. This is primarily due to the K's being dimensional while f is dimensionless, and to their being based on both the Darcy and Fanning frictien factors. The most common friction factor in mining is the Atkinson friction factor, which may be related to the Fanning friction factor (assuming the average weight of a cubic foot of air, at 60°F and atmospheric pressure, is 0.0764 lbf) by

$$f = 3.03\ K\ ,$$

where the K-tables have been tabulated from velocity measurements made in thousands of feet per minute (Bunt, 1960). Hartman (1961), however, relies on the relationship

$$f = 1.214(10)^{7}\ K$$

where the K-tables have been calculated from velocity measurements in feet per minute. Conversion to thousands of feet per minute gives

$$f = 12.14\ K\ ,$$

from which we see that Hartman's friction factor K must be based on the Darcy friction factor. However, he makes no distinction between Darcy and Fanning friction factors, which seems to be a common occurrence in mining literature. The reader is thus again cautioned to be sure of the meaning of the friction factor he/she has obtained.

In determination of the relaxation length, it is necessary to relate the actual Fanning friction factor of the passage to the friction factor of the same passage if the walls were smooth (relative roughness is zero). This has been determined empirically as

$$f[\text{smooth}] = 0.25\left[0.0859\ln\left[\frac{Re}{1.964\ln(Re) - 3.8215}\right]\right]^{-2} \quad (4.64)$$

(Techo, Tickner, and James, 1965; as cited in Chapman and Walker, 1971), which becomes, under the previous assumptions (page 37),

$$f[\text{smooth}] = 0.25\left[0.0859\ln\left[\frac{35886.12w/R}{1.964\ln(35886.12w/R) - 3.8215}\right]\right]^{-2} . \quad (4.65)$$

With the temperature variation and the friction factor determined, the model is now complete and all parameters may be determined.

## The Procedure

The procedure to calculate the microclimatic parameters requires multiple iterations, and is therefore most suited to manipulation using a digital computer. The basic premise

of the model is that there is a smooth transition through the cave from the atmospheric pressure at the lower entrance to the atmospheric pressure at the upper entrance. It is the variation of air velocity that maintains this transition. It is assumed that the stagnation pressure and temperature at the lower entrance are the atmospheric pressure and temperature at that point, since nearly all the energy lost in the drop of pressure and temperature as the air accelerates into the cave is maintained in the kinetic energy of the air (that is, the process is isentropic). Further, the actual pressure at the upper entrance must balance the atmospheric pressure there. Once these boundary conditions are set, calculations may begin.

1) Specify the external conditions: atmospheric pressure at the upper and lower entrances and temperature and relative humidity at the lower entrance (for winter conditions).

2) Specify the cave passage parameters: length, radius, angle of rise, internal wall temperature, and friction coefficient.

3) Calculate the saturated and then the actual specific humidity for atmospheric conditions at the lower entrance.

4) Estimate the initial velocity of the air at the lower entrance.

5) Calculate the actual temperature at the lower entrance and convert to virtual temperature using

the specific humidity for atmospheric conditions.

6) Calculate the initial Mach number.

7) Calculate the initial pressure and density.

8) Calculate the mass rate of flow and the relaxation length.

9) Begin calculating parameters of subsequent cross-sections by calculating for each cross-section the specific humidity, the saturated specific humidity, and the temperature according to the appropriate formulas.

10) Calculate the virtual temperature and iterate for the mach number of each cross-section.

11) Calculate pressure at each cross-section.

12) Continue until end of cave is reached - Is the final pressure equal to atmospheric pressure? - If no, estimate new initial velocity from the temperature and velocity calculated and the <u>actual</u> atmospheric pressure - Return to step 4 - If yes, calculate all remaining parameters from <u>known</u> Mach number humidity, and temperature variations.

Case studies of this and other models are contained in section VI.

## V - INCOMPRESSIBLE MODEL

The compressibility of air severely complicates the determination of speleomicroclimatic parameters as is witnessed by the foregoing model. It would be of great value if air could be treated as incompressible for the purpose of calculating these parameters. This is not an unwarranted assumption for level passages that are short relative to their diameter (the effect of friction on the flow is not extremely strong) if the mach number is small. Shapiro (1953, p.48) states that "...the error produced by neglecting compressibility in the computation of pressure variations is of the order of one-fourth the square of the ratio of the stream velocity to the sound velocity [Mach number]." If the given requirements were met, the assumption of incompressibility should produce an error in the calculation of pressure of roughly only one per cent at a Mach number of 0.2. An error of this magnitude is more than acceptable for the results desired here, but unfortunately the assumptions are not met. With a level passage, no consistent chimney effect can exist, and most caves which exhibit this effect are generally quite long with respect to the size of the passage. The range of Mach numbers that are dealt with here, however, are so low (0.0005 - 0.05) that these may not be sufficient objections, so an incompressible model is worth investigating.

It is obvious that the density of air must vary with

elevation in a non-horizontal passage, so the air cannot be treated as totally incompressible. The assumption being made is that dynamic influences on density are not significant. It is necessary, then, to know how density, or equivalently pressure, varies with elevation under static conditions. Given no flow of air, there can be no temperature differential between the cave air and the surrounding rock, so the internal atmosphere is at the internal wall temperature, TWALL. Since TWALL has been assumed constant throughout the cave, the cave atmosphere (under the assumption of no flow only) is isothermal, and the pressure variation with height may be calculated from the hydrostatic equation

$$dP = -DGdz . \qquad (5.1)$$

Replacing D by P/(Rg)T from equation (4.1) and rearranging, we get

$$\frac{dP}{P} = \frac{-Gdz}{(Rg)T} , \qquad (5.2)$$

which may be integrated to yield

$$\frac{P(2)}{P(1)} = \exp(-G(z(2) - z(1))/((Rg)TWALL)) . \qquad (5.3)$$

Were the temperature variation independent of velocity, the assumption of incompressibility would imply that this equation holds for dynamic as well as static conditions. The temperature variation is strongly dependent upon the air

velocity, however, and hence density is indirectly dependent upon this velocity. The assumption of incompressibility must, in this situation, mean that the actual and stagnation densities are the same, not that the density is invariant. The situation may be more clearly understood by picturing the cave atmosphere as being composed of horizontal layers of infinitesimal thickness. Air density may vary from layer to layer, but is constant within any given layer (Thompson, 1972). The effect of a change in flow conditions, with a resultant change in the temperature distribution, serves not to change the actual densities, but merely to shift the layers upward or downward. It is this upward and downward motion (of the density layers) that allows the cave atmosphere to adjust to external conditions. The solution for the density variation, then, is very similar to that under static conditions, but the temperature variation is now that of equations (4.58), (4.61), (4.62), and (4.63). The solution from equations (4.30) and (5.2) requires numeric integration (trapezoidal rule), since T is no longer constant. This yields

$$D(2) = D(1)\frac{T(2)}{T(1)}\exp\left[\frac{-G\sin\theta}{(Rg)}\left[\frac{1}{T(2)} + \frac{1}{T(1)}\right]\left[\frac{x(2) - x(1)}{2}\right]\right], \tag{5.4}$$

where, as before, $z = \sin\theta x$, so $dz = \sin\theta dx$. Given, then, the temperature variation along the passage, all that remains now is to interpret the boundary conditions in terms

of the flow parameters. In the compressible model, the atmospheric conditions were taken as the initial stagnation conditions, so let us look at equations (4.25) and (4.26), which are

$$To = T\left[1 + \frac{(k-1)M^2}{2}\right], \tag{5.5}$$

and

$$Po = P\left[1 + \frac{(k-1)M^2}{2}\right]^{k/(k-1)}. \tag{5.6}$$

The highest Mach number that we would be concerned with in cave studies is around 0.05, which corresponds, at a temperature of 10°C, to an air velocity of approximately 17 meters/sec. Since T and To are measured in °K, we may generously assume that the range of temperatures of interest to us is 230-310°K, so for To varying within this range, the maximum error in neglecting M is only 0.11 to 0.15°K, a minimal difference. This result was anticipated in the development of equations (5.3) and (5.4), since it was assumed without explanation that the temperature did not vary due to compressibility of the air. For P, however, the power term k/(k-1) is equal to 3.5, and the dependence upon M cannot be ignored. Instead, let us binomially expand the right side of equation (5.6). This gives

$$Po = P\left[1 + \frac{kM^2}{2} + \frac{kM^4}{8} + \frac{k(2-k)M^6}{48} + \ldots\right]$$

$$= P + \frac{PkM^2}{2} + \frac{PkM^4}{8} + \frac{Pk(2-k)M^6}{48} + \ldots$$

$$= P + \frac{PkM^2}{2}\left[1 + \frac{M^2}{2} + \frac{(2-k)M^4}{24} + \ldots\right] . \tag{5.7}$$

But from equation (4.3),

$$\frac{PkM^2}{2} = \frac{PkV^2}{2k(Rg)T}$$

$$= \frac{PV^2}{2(Rg)T} ,$$

and since

$$D = \frac{P}{(Rg)T}$$

from equation (4.1), it follows that equation (5.7) becomes

$$Po = P + \frac{DV^2}{2}\left[1 + \frac{M^2}{4} + \frac{(2-k)M^4}{24} + \ldots\right] . \tag{5.8}$$

Neglecting the Mach number in this equation induces no great error, hence we arrive at

$$Po = P + \frac{DV^2}{2} , \tag{5.9}$$

which is familiar to all those acquainted with fluid dynamics as the expression for the stagnation pressure for incompressible flow. P is often called the static pressure

and $DV^2/2$ the velocity pressure. The stagnation pressure, in this context, is often called the total pressure for obvious reasons.

The solution is now available from the Bernoulli - Euler equation

$$0 = dP + DVdV + DG\sin\theta dx + \frac{fDV^2dx}{R} \tag{5.10}$$

by integration over the length of the cave. This yields (using Simpson's one-third rule),

$$\begin{aligned} 0 = \; & P(N+1) - P(1) + D(1)V(1)(V(2) - V(1)) \\ & + G\sin\theta(D(1) + 4D(2) + \ldots + D(N+1))(x(2) - X(1))/3 \\ & + fV(1)((1/D(1)) + (4/D(2)) + \ldots + (1/D(N+1)) \\ & \quad \cdot (x(2) - x(1))/(3R) \end{aligned} \tag{5.11}$$

since DV is constant when w and R are constant. Rearranging and noting that

$$P(1) = PEXTL - 0.5D(1)V(1)^2 ,$$

we get

$$\begin{aligned} & P(N+1) \\ = \; & PEXTL - 0.5D(1)V(1)^2 - D(1)V(1)^2((D(1)/D(2)) - 1) \\ & - G\sin\theta(D(1) + 4D(2) + \ldots + D(N+1))(x(2) - X(1))/3 \\ & - fV(1)((1/D(1)) + (4/D(2)) + \ldots + (1/D(N+1)) \\ & \quad \cdot (x(2) - x(1))/(3R) \end{aligned} \tag{5.12}$$

From this, the initial velocity V(1) may be derived by

iteration, comparing P(N+1) to PEXTU until they are sufficiently close (so that each calculated velocity is within 0.0001 m/sec of the previously calculated velocity). With the temperature variation obtained from equations (4.58), (4.61), (4.62), and (4.63), the solution is now readily available from equations (5.4) and (5.9), and the equations for temperature distribution, (4.58), (4.61), (4.62) and (4.63). The procedure is as follows

1) Specify the external atmospheric conditions.
2) Specify the cave parameters.
3) Estimate the initial velocity and calculate the temperature and hence pressure variation step by step along the passage.
4) From the external pressure and calculated air temperature at the <u>upper</u> entrance, calculate the pressure at the lower entrance.
5) Compare the calculated pressure at the lower entrance with the stagnation pressure (atmospheric) there, and estimate the new initial velocity.
6) Continue the iteration on the initial velocity until the desired accuracy is reached.
7) Calculate all remaining parameters.

The above procedure has been simplified by the ignoring of moisture, and the reader who wishes a more complete procedural description should compare the above with that for the compressible model on pages 48 and 49.

## Unconstrained Temperature Profile

A further simplification which can be made to the incompressible model without severe loss of accuracy is suggested by Wigley and Brown (1971). Removal of the constraint that the air may not become saturated produces only minor error and allows the use of equations (4.58) and (4.62) for the temperature variation along the entire passage, thus avoiding the iterative solution of equation (4.61). "[One is] justified in using the non-constrained solution for the temperature distribution in the majority of quantitative applications" (Wigley and Brown, 1971, p.312). The procedure here is identical to that in the previous section. Equation (5.12) again used to iterate for the initial velocity, with the temperature variation coming from equations (4.58) and (4.62), depending on whether or not the effects of moisture are considered. A comparison of this and the previous models is found in the following section.

## VI - CASE STUDIES

In order to compare the three previously described models, the following hypothetical situation is constructed. Assume that the ideal dynamic cave described in section IV is 1000 metres long and rises at a constant angle of 10°. The internal wall temperature is 10°C, the wall friction factor is 0.030, and the radius of the passage is 1.5 metres. The walls of the passage will be considered wet throughout its length, so E = 1.0 . In order to determine the internal microclimatic conditions, all that is required are the external atmospheric conditions. Three cases were devised which, for simplicity in the comparison, vary only in external temperature. These three cases are:

| | | | |
|---|---|---|---|
| Atmospheric pressure at upper entrance | | = | 925 mb |
| Atmospheric pressure at lower entrance | | = | 950 mb |
| Relative Humidity at lower entrance | | = | 0.50 |
| Temperature at lower entrance | (a) | = | -20 °C |
| | (b) | = | -10 °C |
| | (c) | = | 0 °C |

The internal parameters were calculated at 25 metre intervals and, for simplicity, the variation of the specific humidity (at saturation) with pressure was ignored. The results are tabulated in Appendix A, while the computer programs used are found in Appendix B.

## The Compressible Model

As this study is entirely theoretical in nature, the norm by which models are accepted or rejected must of necessity be that one which makes the fewest assumptions about the nature of the flow, namely the compressible model. The three cases studied all show very little variation between density, pressure, and temperature and their related stagnation properties. The difference is on the order of 0.05 per cent of the stagnation properties and hence may generally be ignored. The reason for this minute difference is evident from the Mach numbers calculated. In no instance is the Mach number greater than 0.015, and the variation along the passage is minimal. Thus the relationship between velocity and the state parameters may be ignored (with the obvious exception of temperature, which depends on the rate of heat transferral to or from the air, a velocity dependent operation).

The relative humidity bears special note, for it almost immediately reaches 100 per cent. This is to be expected, however, for under winter conditions such as those here, there is very little water vapor in the air at saturation. Upon entering the cave, the air very quickly gains moisture from the cave walls and saturation is achieved almost at once. The opposite effect occurs in summer for the upper entrance, where the air is capable of holding considerably more water vapor than under winter conditions. Upon entering

the cave, the air takes on moisture from the walls for a much longer period of time (than under winter conditions with the _same_ relative humidity at the entrance drawing air) before saturation is finally (if ever) reached (if the external air is saturated, however, then the entire cave will also be saturated, regardless of season).

By far the most curious result obtained from the compressible model, however, is the air velocity. While the overall magnitude of the velocity in each case is reasonable (in light of the observed magnitude of most cave winds) considering the simplifications imposed (no constrictions, no bends, etc.), the relative magnitude _between_ cases appears contradictory. As the external temperature approaches the internal wall temperature, the internal-external pressure differential decreases, so the air velocity should also decrease. Exactly the opposite, however, is found in the three cases studied; as the external temperature increases from -20°C to -10°C, the initial velocity increases from 4.25 m/sec to 4.42 m/sec to 4.57m/sec. The contradiction is only apparent, however, and is due entirely to the cases chosen. Because the external atmospheric pressure at both entrances was specified as constant, an increase in temperature at the lower entrance must bring about a _decrease_ in air density there. Thus, to maintain the designated pressure differential between the entrances, the less dense air must move faster to have the equivalent effect. The reader should thus be aware that the

cases under study are not meant to be related in any manner through time, but rather are distinct situations in which certain variables are held constant so that the others may be more fully understood. In an actual situation, one would expect some variation in pressure to accompany such a variation in temperature.

## The Incompressible Models

Comparison of the results generated by the incompressible models with those produced by the compressible model shows excellent agreement with regard to pressure variations. The difference between all models in this respect, is again on the order of 0.02 per cent, but this could easily be anticipated. The pressure differential between the entrances is only 25 millibars and furthermore, the entrance pressures are *fixed* limits between which each model *must* operate. Considering that the printed results are accurate only to 0.1 millibars, one would readily expect the pressure variation along the passage to differ little from a mean of 2.5 millibars per hundred metres, and such is in fact the case. The variation over all cases and all models is 2.2 to 2.6 with the variation consistently greater at the lower entrance and less at the upper. The predictive ability of the incompressible models, then, cannot be judged on their ability to predict pressure variations.

When temperature and density are considered,

differences between the models begin to appear. The constrained incompressible model still agrees well with the compressible, the variation of the former over the latter being on the order of 0.2 per cent. The unconstrained incompressible model, however, varies nearly 3.0 per cent from the compressible model in both temperature and density. While errors of this magnitude might not appear to be significant in light of the numerous simplifications used to generate them, it should be noted that for the analysis of error, temperatures must be expressed in degrees Kelvin and a 3.0 per cent error at the temperatures under consideration is approximately 8.0°C. This difference is most readily seen in case 1, where the temperature at 150 metres is calculated at -8.46°C in the unconstrained incompressible model and at -1.44°C in the compressible model. While Wigley and Brown (1971) have shown that, at the same velocity, and with a relatively small temperature differential between the atmosphere and the cave wall, the constrained and unconstrained temperature profiles are sufficiently similar, here the temperature differential is extreme (30°C) and markedly erroneous estimations of temperature are produced.

In general, the constrained incompressible model agrees well with the compressible model. Temperature, pressure, and density vary less than 0.5 per cent from the compressible values, while velocity varies on the order of 3.2 per cent. In nearly all applications, then, the constrained incompressible model presents an excellent approximation to

the compressible.

A word of caution concerning velocity estimations from the above models is in order. Wigley and Brown (1971) have pointed out that the relaxation length for temperature is very insensitive to changes in velocity (the dependence is only to the 0.2 power). The pressure variation is also weakly dependent upon velocity, for it was noted during the iterations for initial velocity that a reduction of only two millibars in the calculated pressure at the upper entrance required an increase in velocity of nearly 1.0 metres per second. Very accurate measurements or estimates of the pressure differential between the entrances would thus be necessary for the reasonable estimation of velocities within the cave.

The results of the three case studies are displayed graphically in figures 11 to 20, and the actual computer print-out and the programs used appear in Appendices A and B respectively.

## VII - CONCLUSION

### Context and Contribution of Present Study

It has been shown that the constrained incompressible model is an excellent approximation to the compressible model for the flow of air in caves. At no point in the case studies did the two differ beyond the bounds of the error inherent in the numerical solutions, and there is no reason to expect any greater difference if these solutions were refined. Despite the long distances over which friction works, and despite the substantial changes in elevation, it is clear that the mach number is sufficiently low to allow use of a modified incompressible formulation. The Bernoulli-Euler equation in its compressible form, the hydrostatic equation, and the temperature variation derived by Wigley, and Brown (1971) give a more than adequate determination of the microclimatic variables in our ideal cave.

Any extension of this model to actual caves must be done with caution, however. The reader is reminded that the present study is an entirely theoretical development in which numerous simplifying assumptions have been made. In relatively short, unconstricted, non-sinuous caves with little vertical extent, the present theory may be expected to yield reasonable results. If these conditions are not met, the agreement of the model with reality may be poor. The effects of sinuosity may be accommodated by the friction

factor, for the effect of bends in the passage is to resist the flow of air. Changes in cross-sectional area, however, would be somewhat more bothersome. Their inclusion into the compressible model's equation for mach number variation (4.32) is straightforward. Wigley and Brown (1971), however, have pointed out that their inclusion into the equations for temperature variation makes the relaxation length variable and may negate the empirical relationship between the Nusselt, Prandtl, and Reynolds numbers. Therefore, given the present state of heat and mass transfer theory, an assumption of constant radius (i.e., an average value) may be necessary.

In actual caves, changes in elevation and, more important, changes in depth beneath the surface produce changes in the rock temperature, making TWALL non-constant. The requisite differential equations may be integrated numerically assuming a non-constant wall temperature, but again the empirical relationship between the Nusselt, Prandtl, and Reynolds numbers may not hold. From the study of Kays (1966), it would appear that this relationship may still be a reasonable approximation, but a more thorough knowledge of heat and mass transfer theory than the author possesses would be necessary to pursue this point.

In sum, then, this thesis analyses the problem and the constraints involved, rather than presenting a solution as such to that problem. The modified incompressible model

presented is only a basic framework. Refinements to the theory are needed but, more important, substantial fieldwork is called for to adapt this model to reality. Only through well-planned empirical studies may the theory more closely approximate actuality.

## Future Research

Concurrent with the wide acceptance of static theories for cave air flow is a lack of carefully executed fieldwork. The author knows of no instance where the air temperature, pressure, and velocity within a cave have been measured continuously over a long period of time at more than one location. The complete monitoring of a cave at numerous locations along its length needs to be undertaken, but there are few caves in Canada that are sufficient for such studies. Nakimu Caves in Glacier National Park, British Columbia, may be adequate, but the presence of a substantial stream in the cave complicates the temperature profile and alters the air velocity through entrainment. Grotte Valerie, Northwest Territories, has also been suggested, but it is over 70 miles from the nearest highway and thus presents considerable problems in logistics. Certain caves on Vancouver Island, British Columbia, may also suffice, though they tend to be too short or too constricted. Nevertheless, the behavior of these caves under changing external conditions needs to be monitored carefully to shed light on the empirical relationships between the numerous parameters

involved.

The extension of the steady-flow case to unsteady situations needs also to be undertaken. Wigley and Brown (1971) have attempted to predict temperature profiles under changing external conditions, but they assume a constant velocity, which can at best be considered only an approximation. The solution of the unsteady case is of great importance for the use of airflow in caves in the remote sensing of subsurface karst characteristics, as well as in the determination of variability of the subterranean atmosphere. It is this combination, then, theory and empiricism, that will ultimately lead to a better understanding of subterranean microclimatic conditions.

Table I: SYMBOLISATION AND UNITS

| Symbol | Units | Meaning |
|---|---|---|
| A | $m^2$ | cross-sectional area of passage |
| A[wall] | $m^2$ | area of passage wall |
| B | m | hydraulic diameter |
| c[p] | $m^2/sec^2-^oK$ | specific heat of dry air at constant pressure |
| D | $kg/m^3$ | mass density |
| Do | $kg/m^3$ | stagnation density |
| dh[pr] | $m^2/sec^2$ | change in enthalpy for change in state |
| E | - | fraction of passage wall that is moist |
| f | - | Fanning friction factor |
| f[smooth] | - | friction factor for smooth passage |
| G | $m/sec^2$ | acceleration due to gravity |
| H | $m^2/sec^2$ | latent heat of evaporation (or sublimation) |
| h | $m^2/sec^2$ | specific enthalpy |
| ho | $m^2/sec^2$ | stagnation enthalpy |
| k | - | ratio of specific heat constant pressure to specific heat constant volume |
| L | $^oK/m$ | dry adiabatic lapse rate |
| M | - | mach number - fraction of speed of sound |
| Nu | - | Nusselt number |
| P | $kg/m-sec^2$ | pressure |
| Po | $kg/m-sec^2$ | stagnation pressure |

| | | |
|---|---|---|
| PEXTL | kg/m-sec$^2$ | atmospheric pressure at lower entrance |
| PEXTU | kg/m-sec$^2$ | atmospheric pressure at upper entrance |
| Pr | - | Prandtl number |
| Q | m$^2$/sec$^2$ | heat per unit mass of air |
| q | - | specific humidity |
| QEXT | - | specific humidity at lower entrance |
| QSAT[T] | - | specific humidity at saturation for temperature T |
| QSPEC | - | mean specific humidity of cave air |
| QWALL | - | specific humidity at saturation for temperature TWALL |
| R | m | radius of passage |
| Re | - | Reynolds number |
| Rg | m$^2$/sec$^2$-$^o$K | gas constant for dry air |
| Rv | m$^2$/sec$^2$-$^o$K | gas constant for water vapor |
| T | $^o$K | temperature |
| t | sec | time |
| t[wall] | kg/m-sec$^2$ | wall shear stress |
| To | $^o$K | stagnation temperature |
| Tp | $^o$K | potential temperature |
| TEXTL | $^o$K | temperature at lower entrance |
| TSAT | $^o$K | temperature at XSAT |
| Tv | $^o$K | virtual temperature |
| TWALL | $^o$K | wall temperature in cave |
| V | m/sec | air velocity |
| w | kg/sec | mass rate of flow |

| | | |
|---|---|---|
| w | hrs. | wavelength of temperature variation |
| X | $kg\text{-}m/sec^2$ | component of gravitational force working against flow |
| x | m | distance into cave |
| xo | m | relaxation length - distance at which temperature differential has decayed to 1/e its original value |
| xsat | m | distance into cave at which saturation is achieved |
| Z[x] | m | elevation of point in cave as a function of x |
| Z | m | elevation above lower entrance; depth below surface |
| $\theta$ | - | angle of inclination of passage |
| (i) | - | denotes value of variable corresponding to cross-section at distance x(i) into cave |

FIGURE 1: Idealised Static Cave

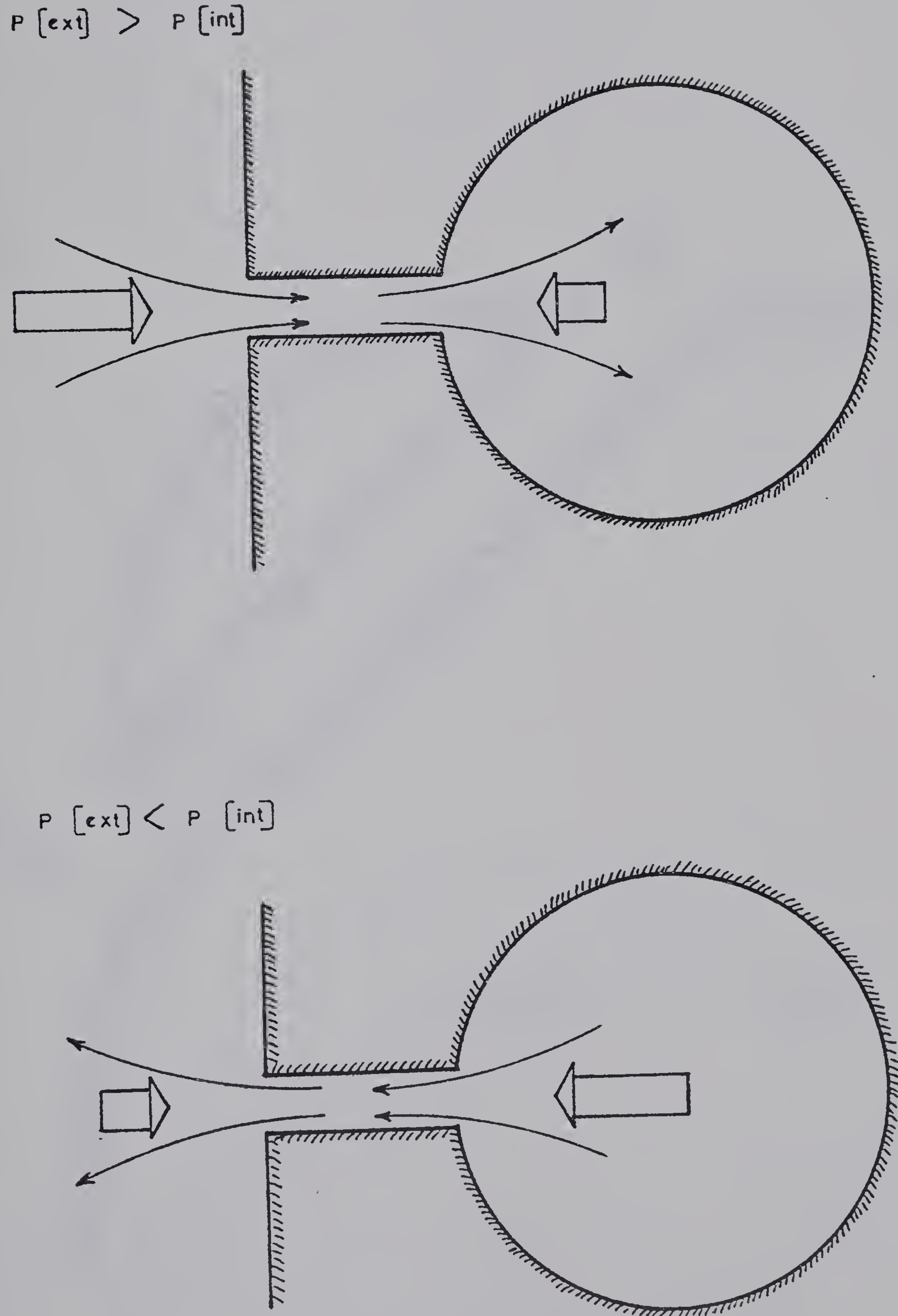


FIGURE 2: Dynamic Cave in Equilibrium with Atmosphere
(after Cigna, 1963)

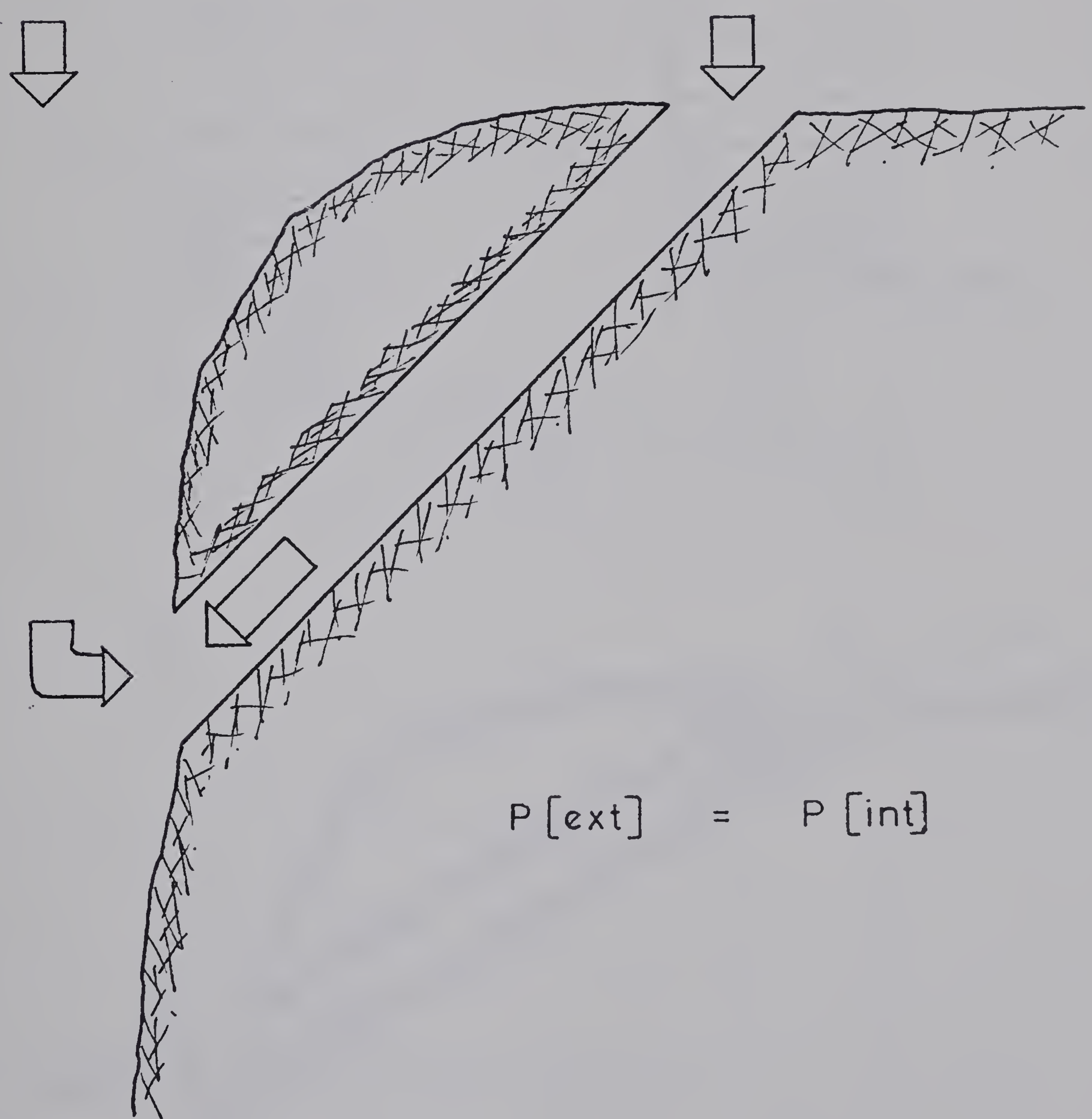
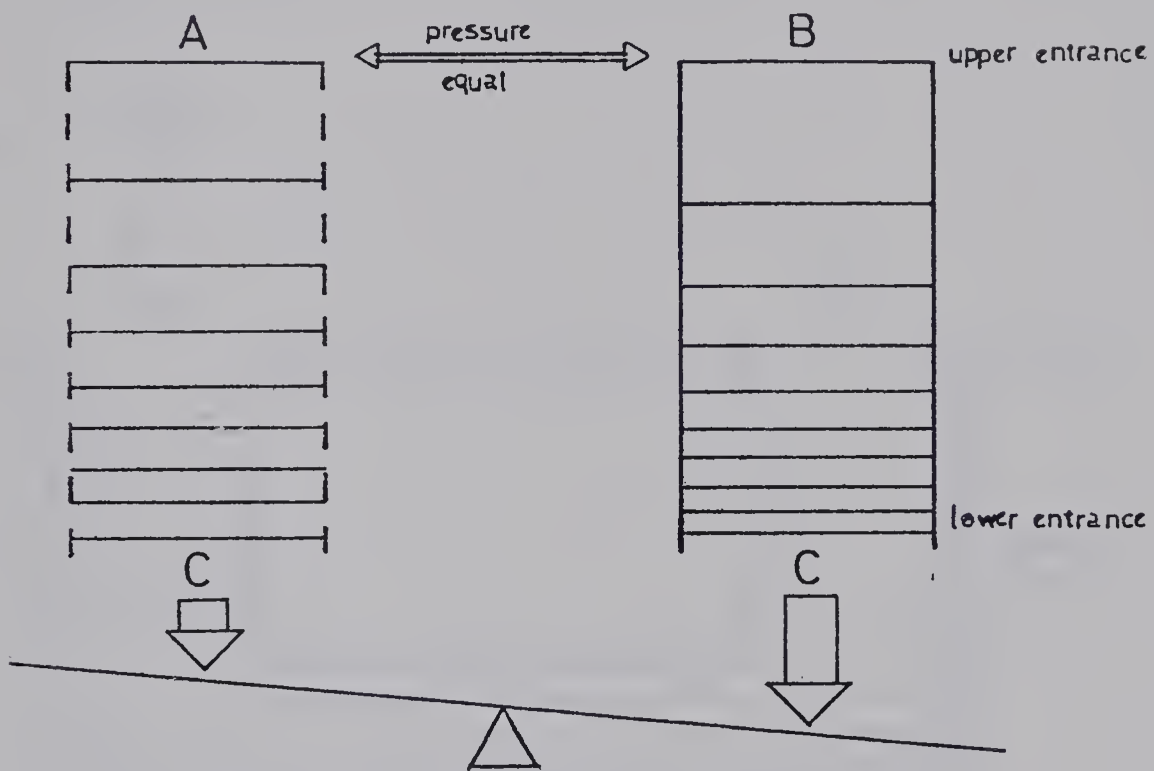


FIGURE 3: Air Columns of Different Weights
- Summer Conditions in a Dynamic Cave



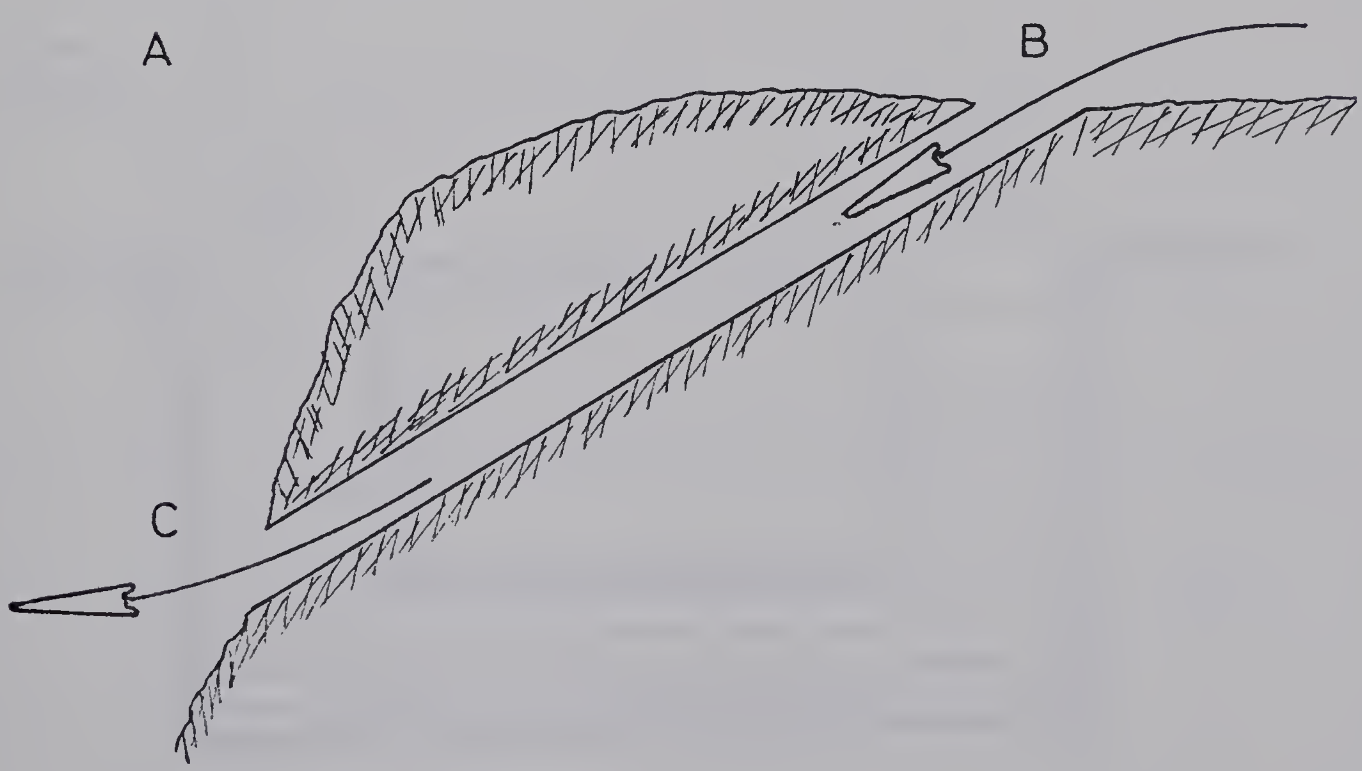
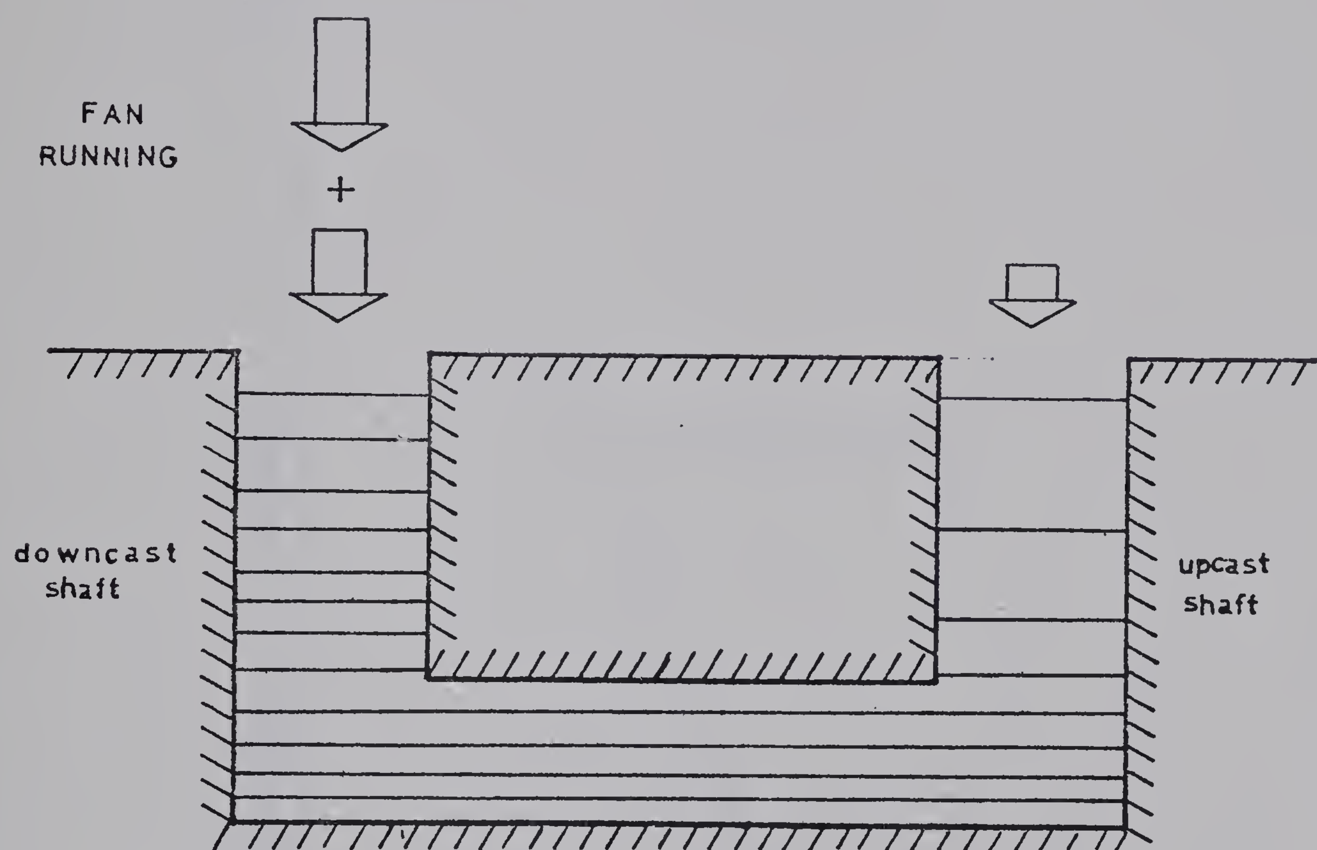



FIGURE 4: Mine with Artificial Ventilation - Winter Conditions (Enhanced Natural Ventilation)



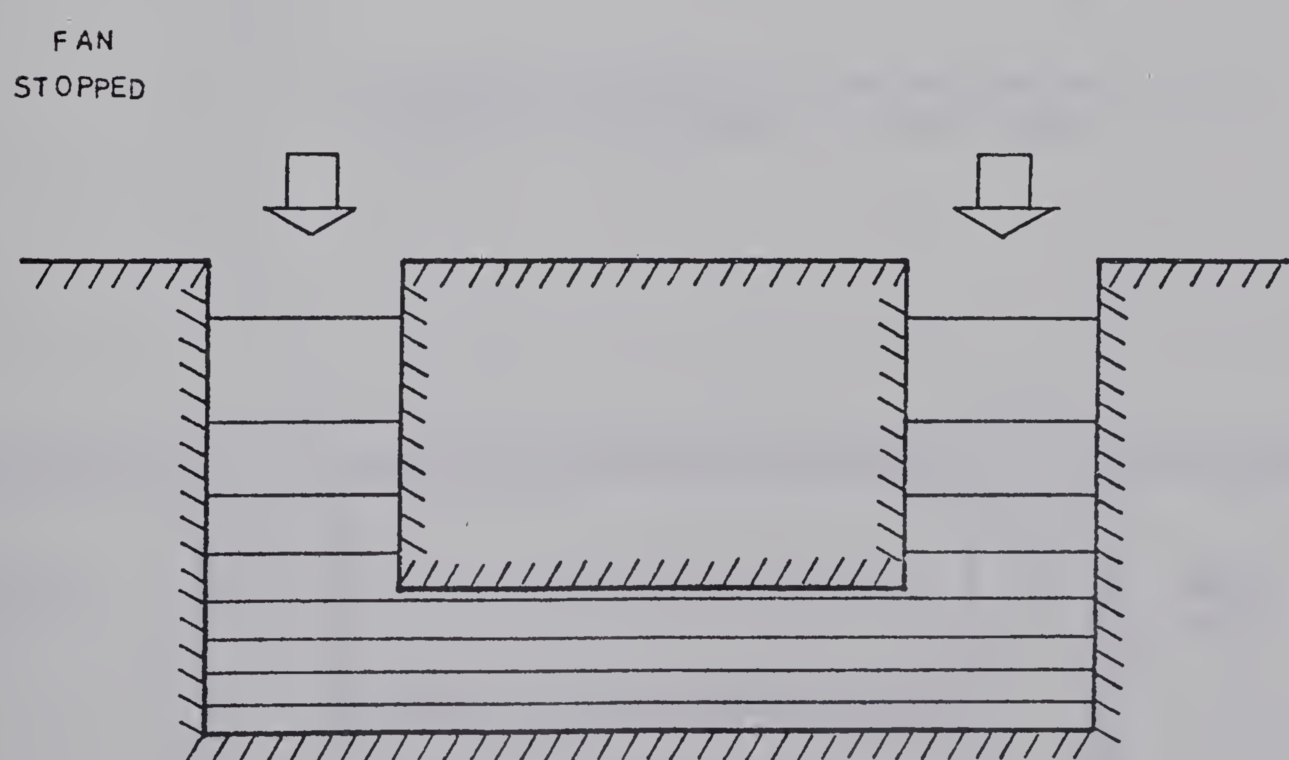
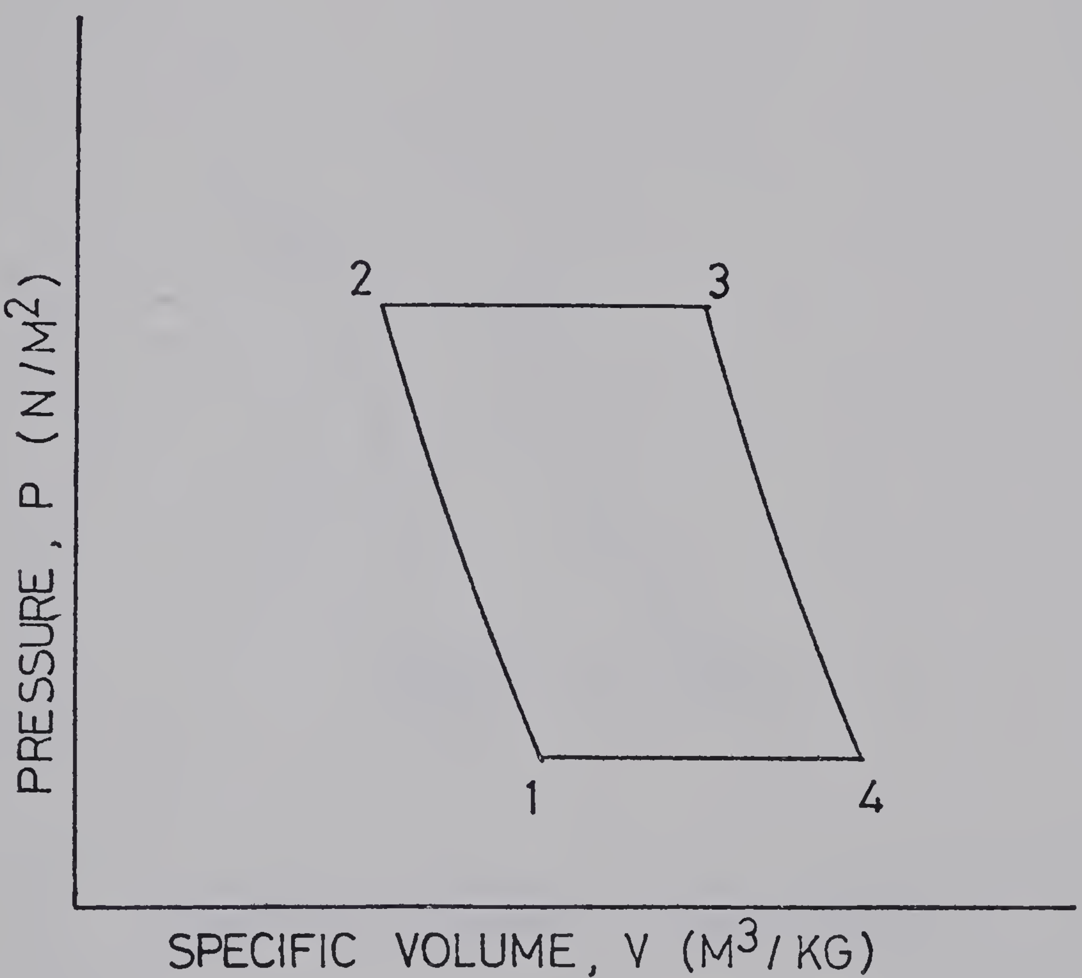



FIGURE 5: IDEAL PRESSURE-SPECIFIC VOLUME GRAPH (THE JOULE OR BRAYTON CYCLE (AFTER HALL, 1967)



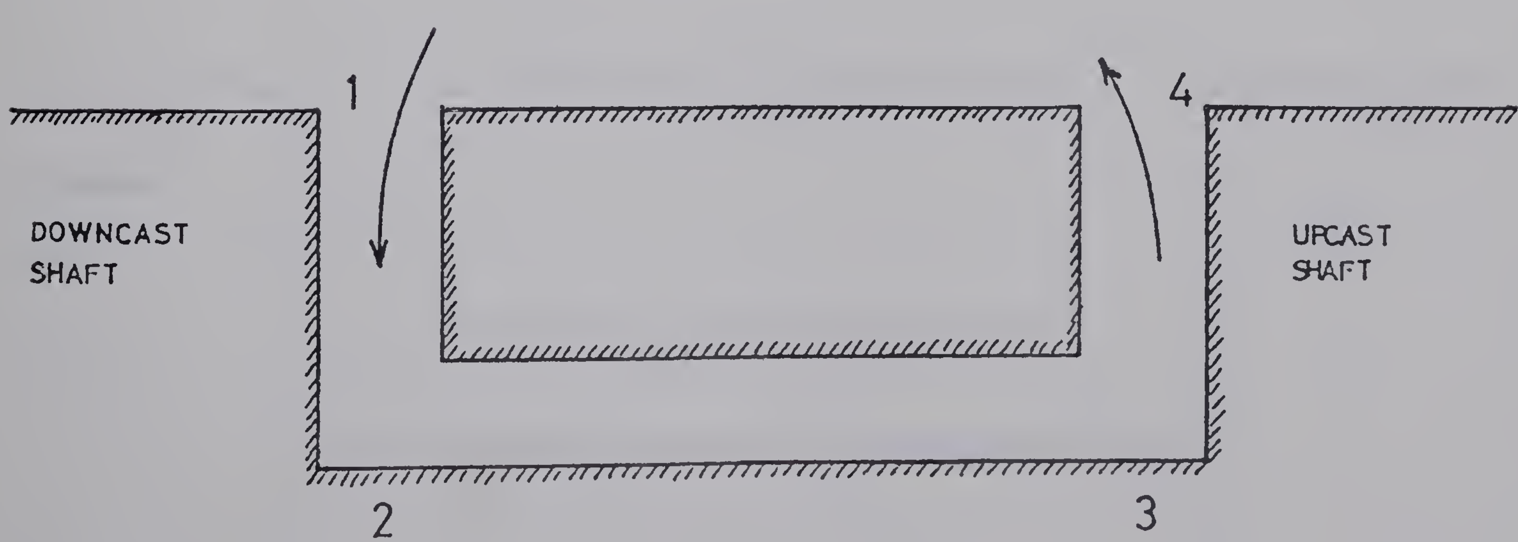
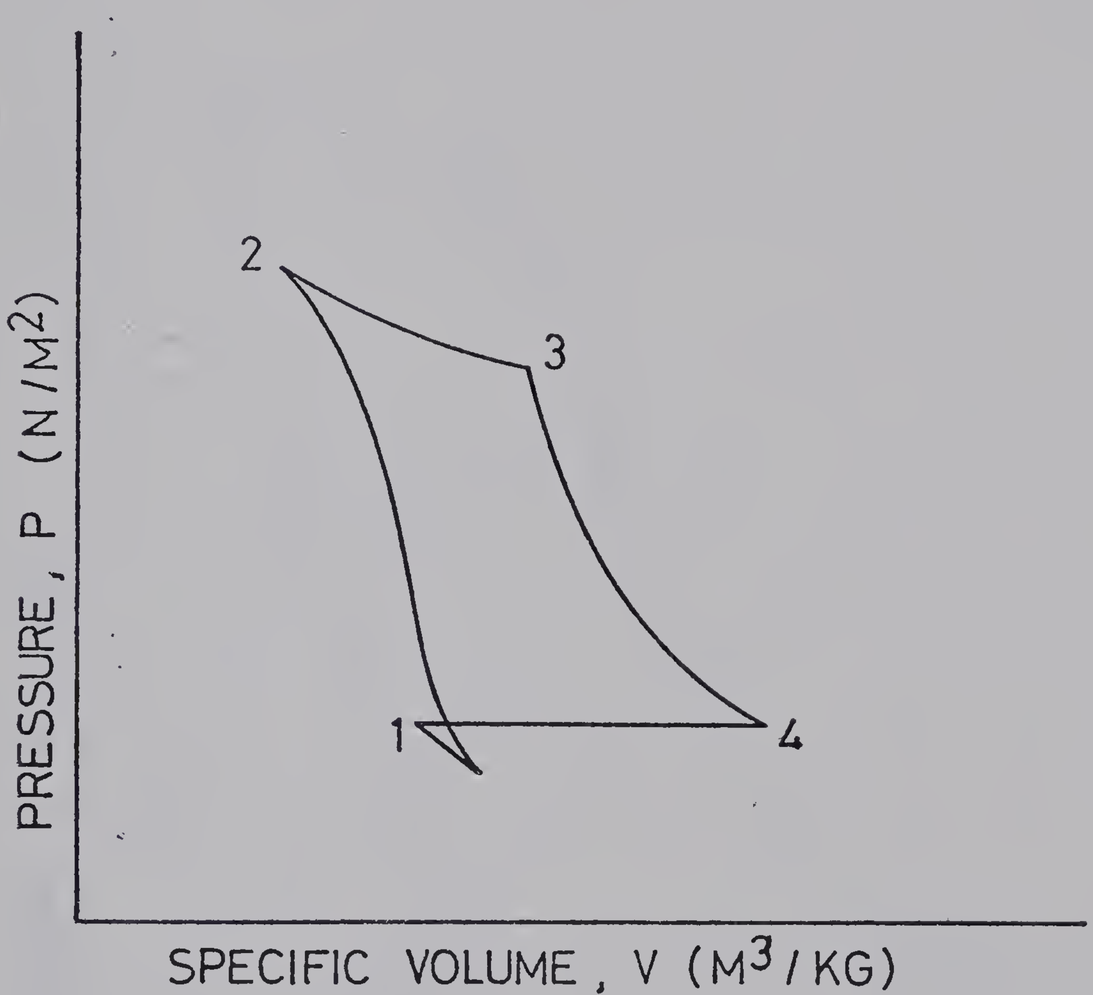



FIGURE 6: Actual Pressure - Specific Volume Graph (after Hall, 1967)



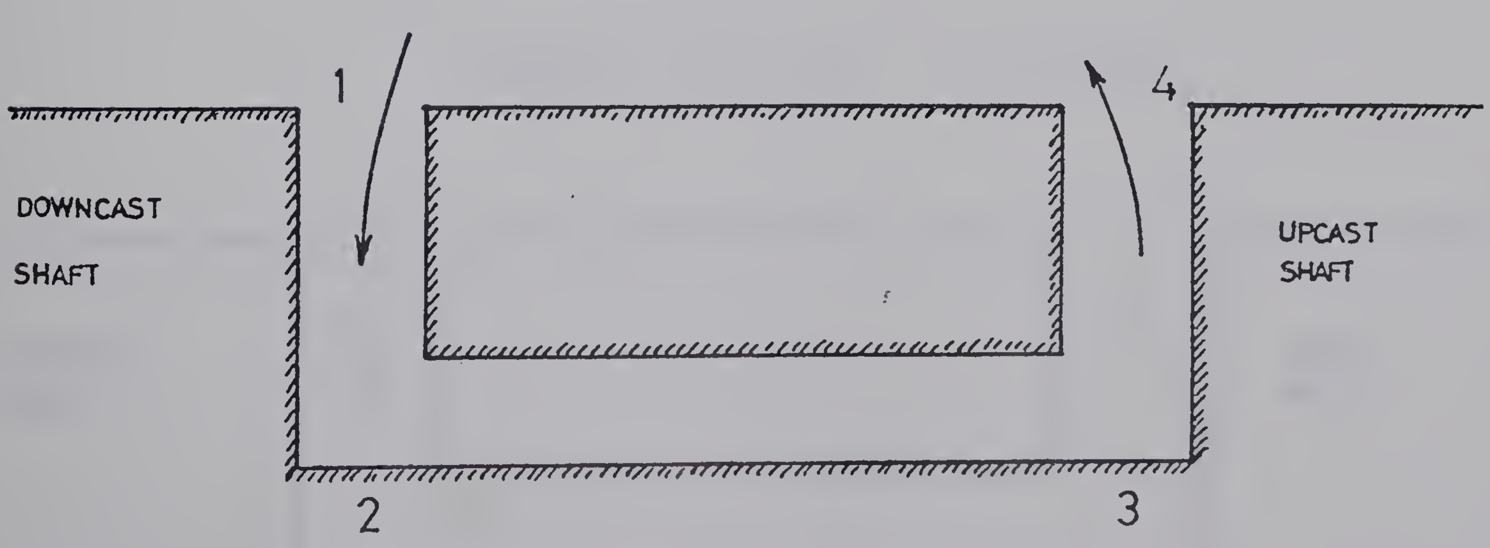



FIGURE 7: Pressure - Specific Volume Graph for Step-by-Step Analysis (after Hall, 1967)

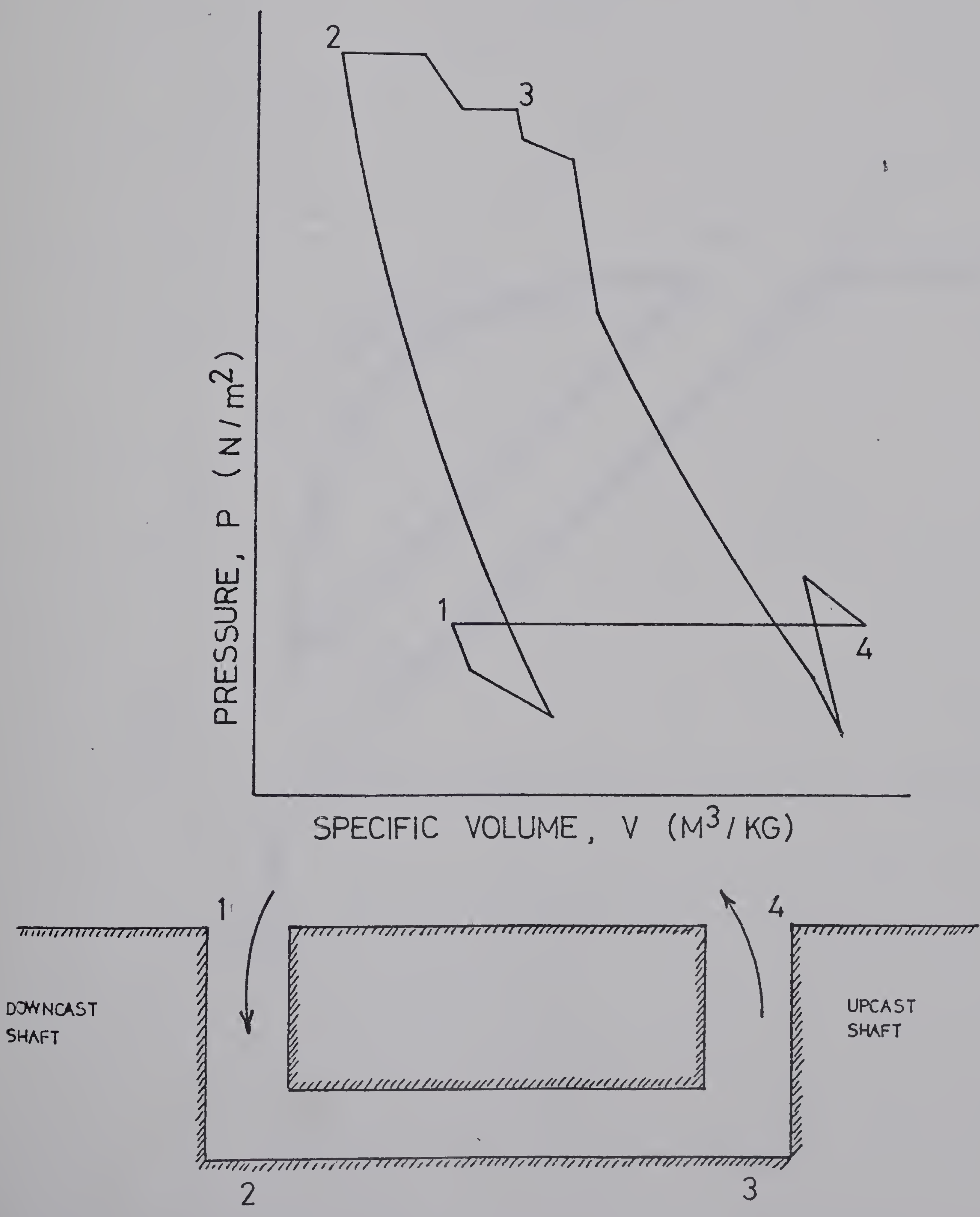


FIGURE 8: Ideal Dynamic Cave
(see Table 1 for explanation of symbols)

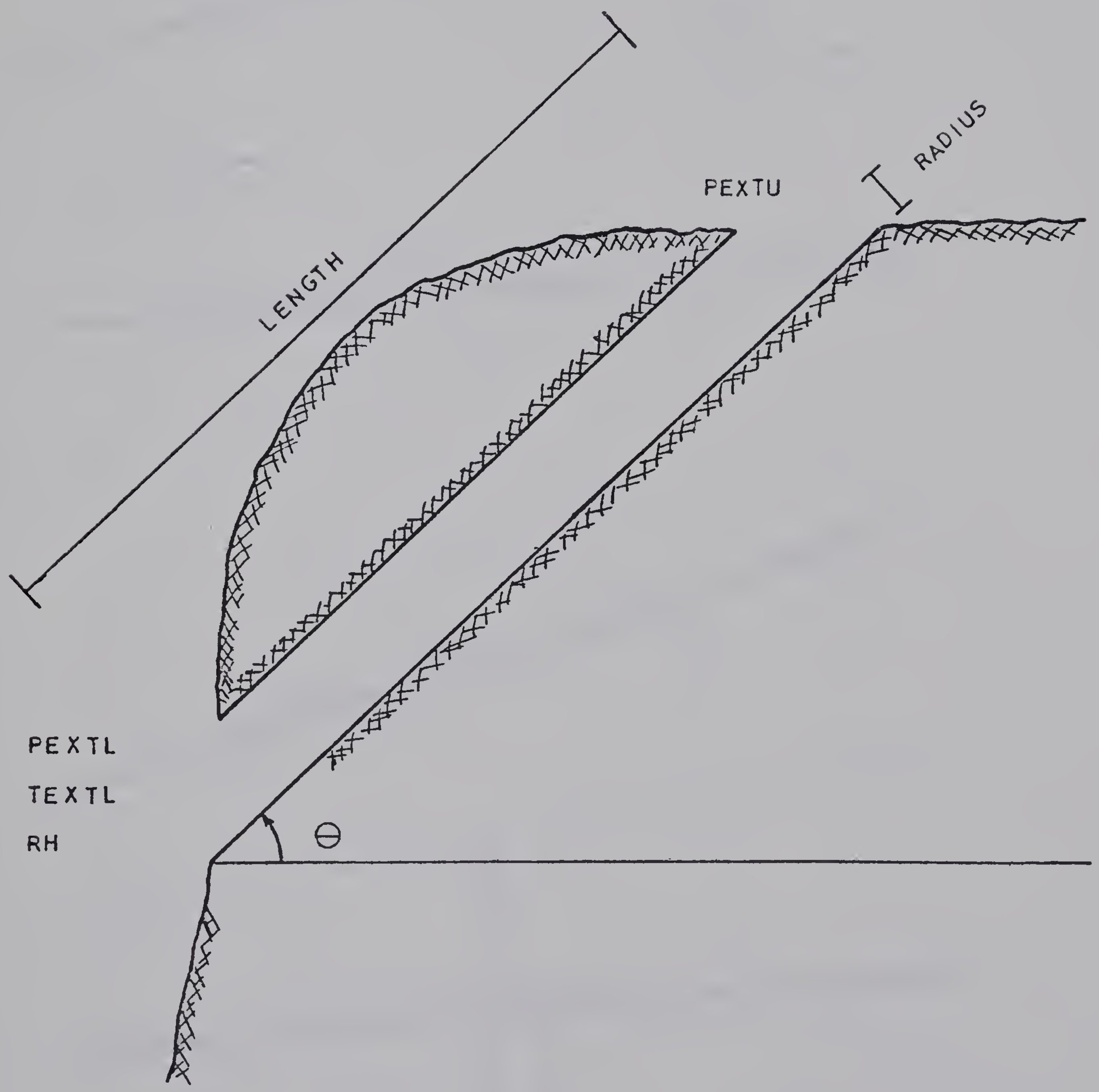
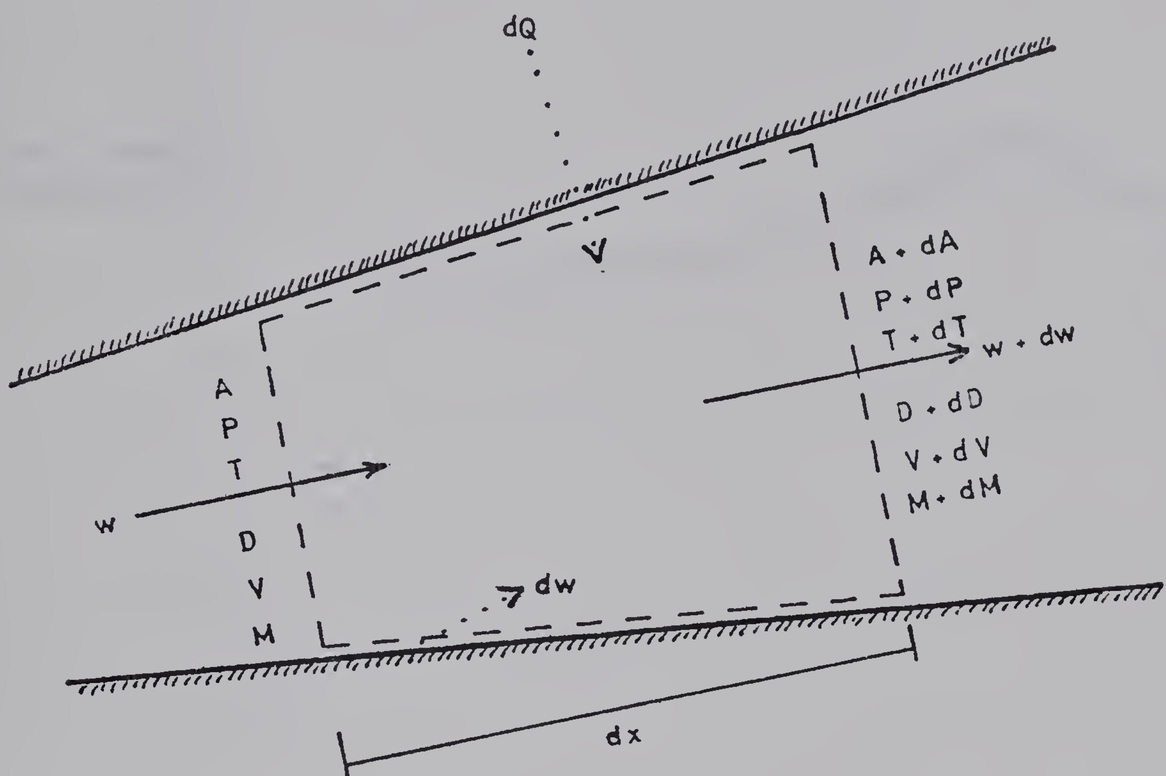


FIGURE 9: The Control Volume (after Shapiro, 1953)



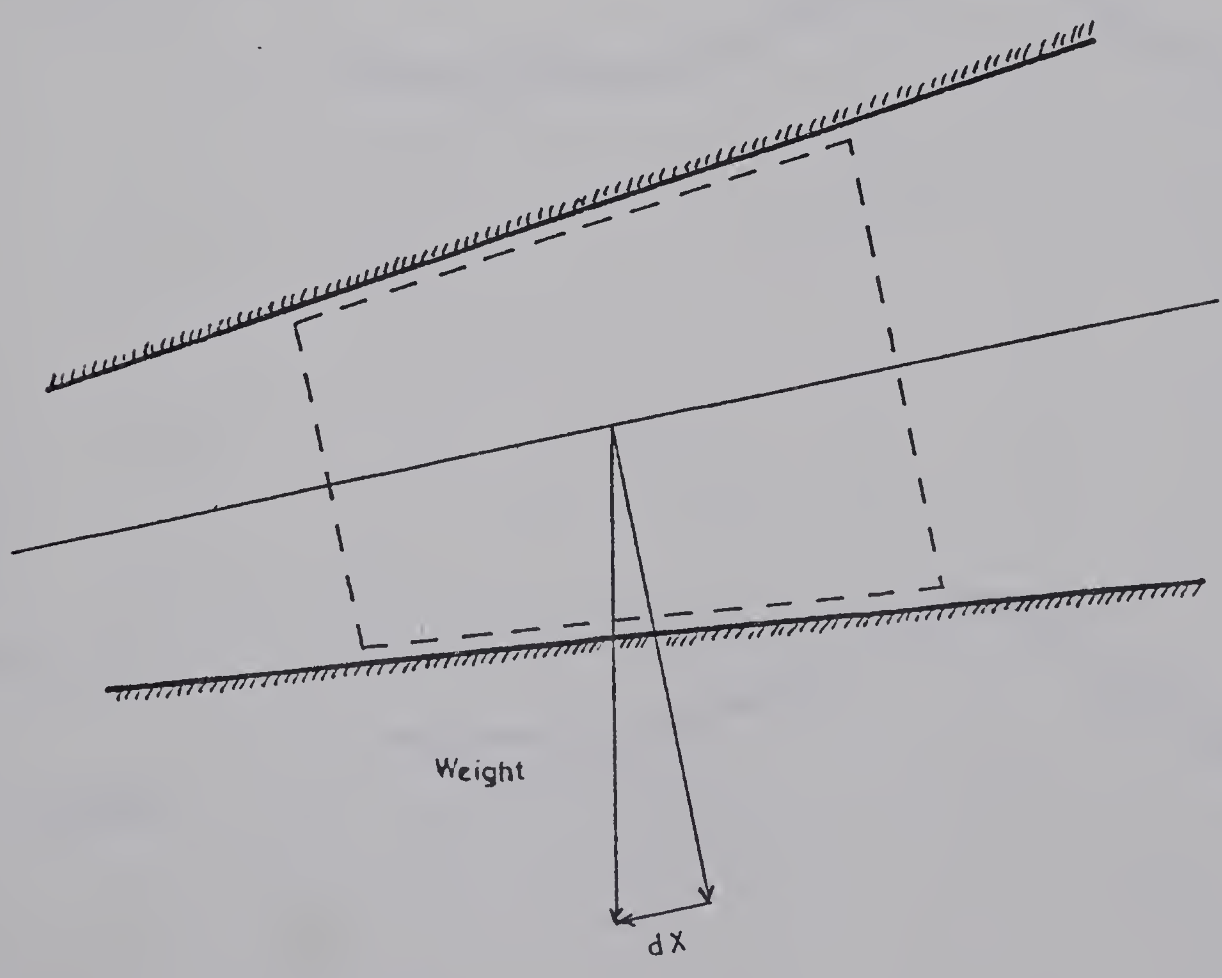
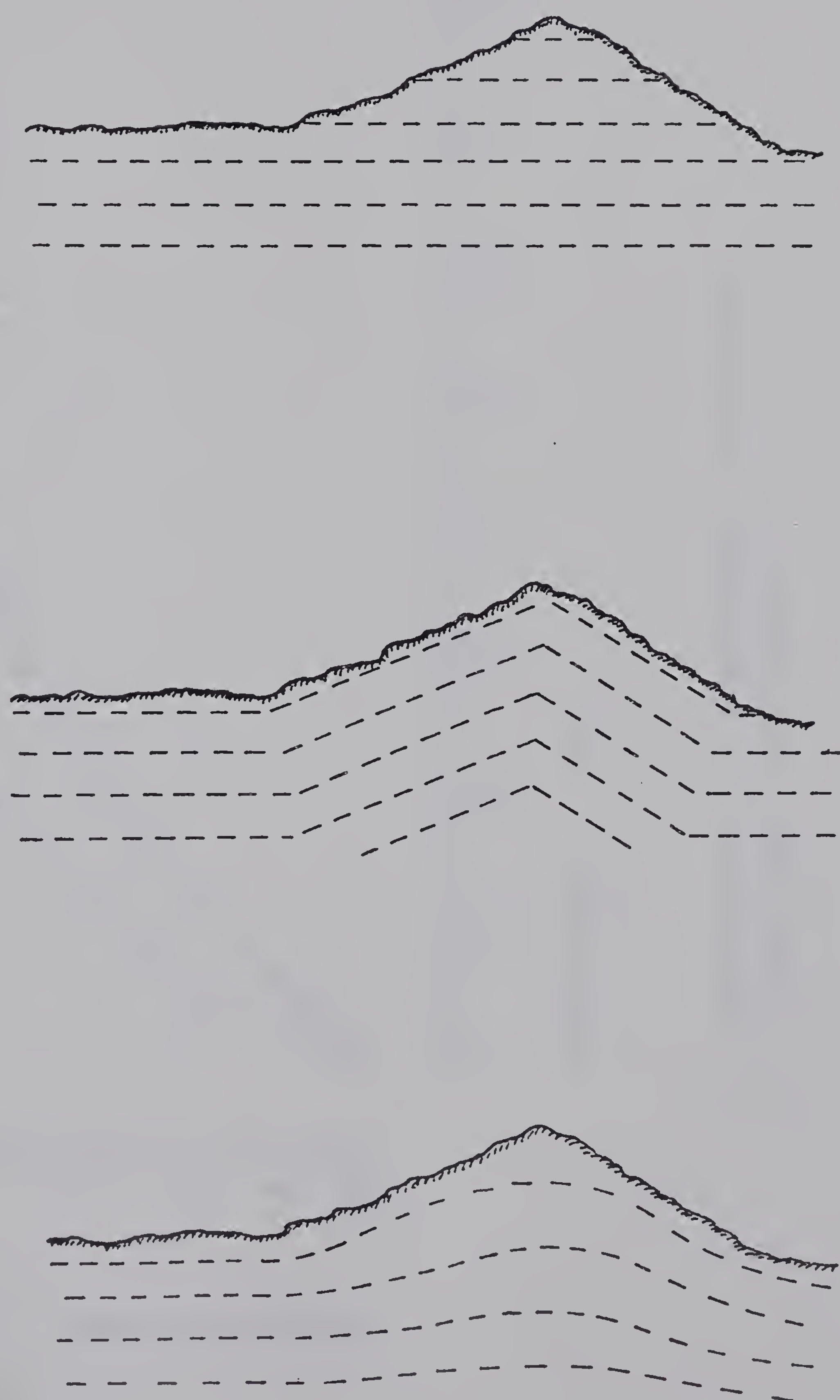



FIGURE 10: Approximation to Geothermal Gradient

a) LEVEL SURFACE ASSUMPTION

b) NORMAL TO SURFACE ASSUMPTION

c) ACTUAL



— — — Isotherms

FIGURE II: Temperature - Case I

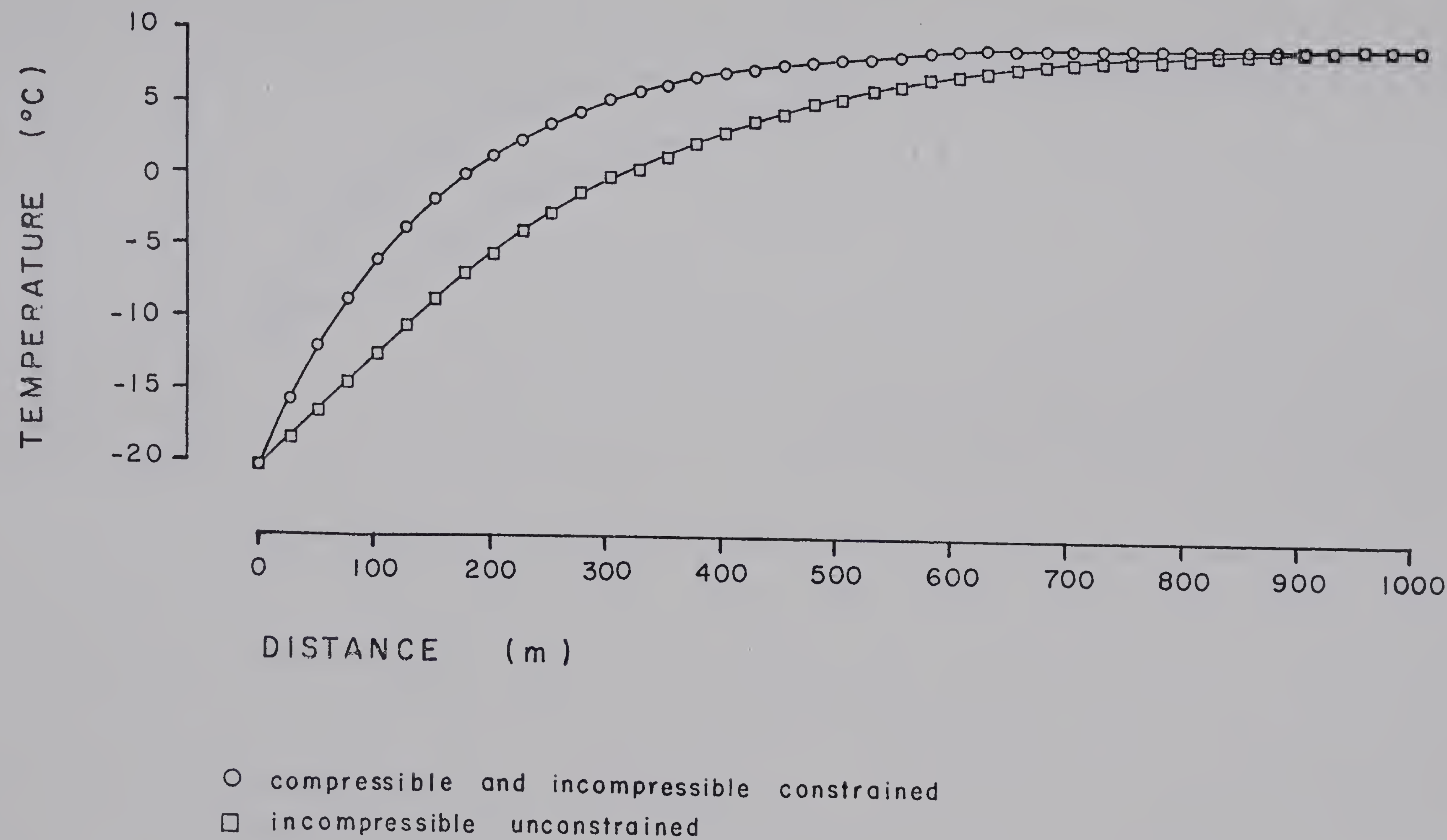


FIGURE 12: Temperature - Case 2

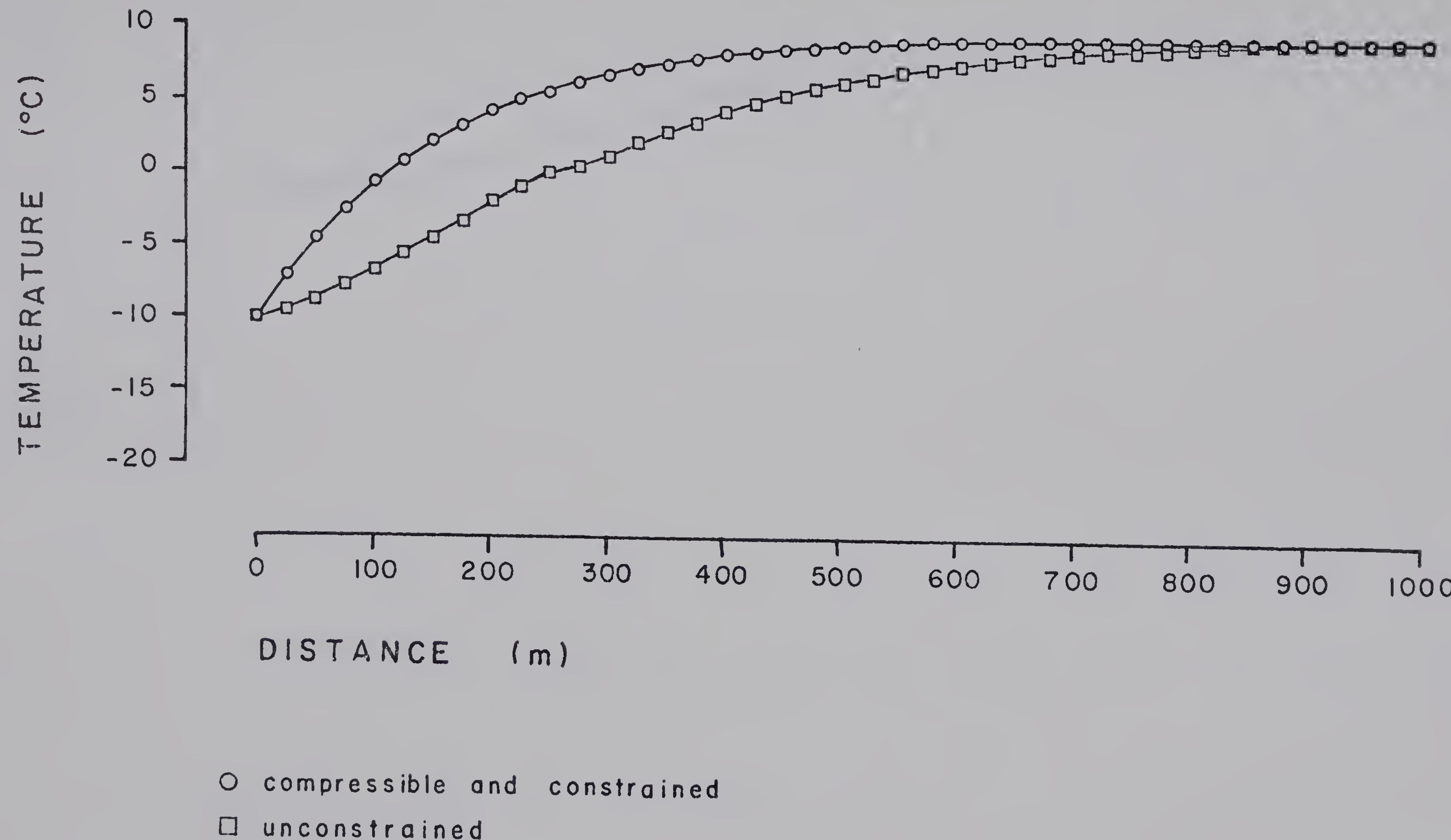


FIGURE 13: Temperature - Case 3

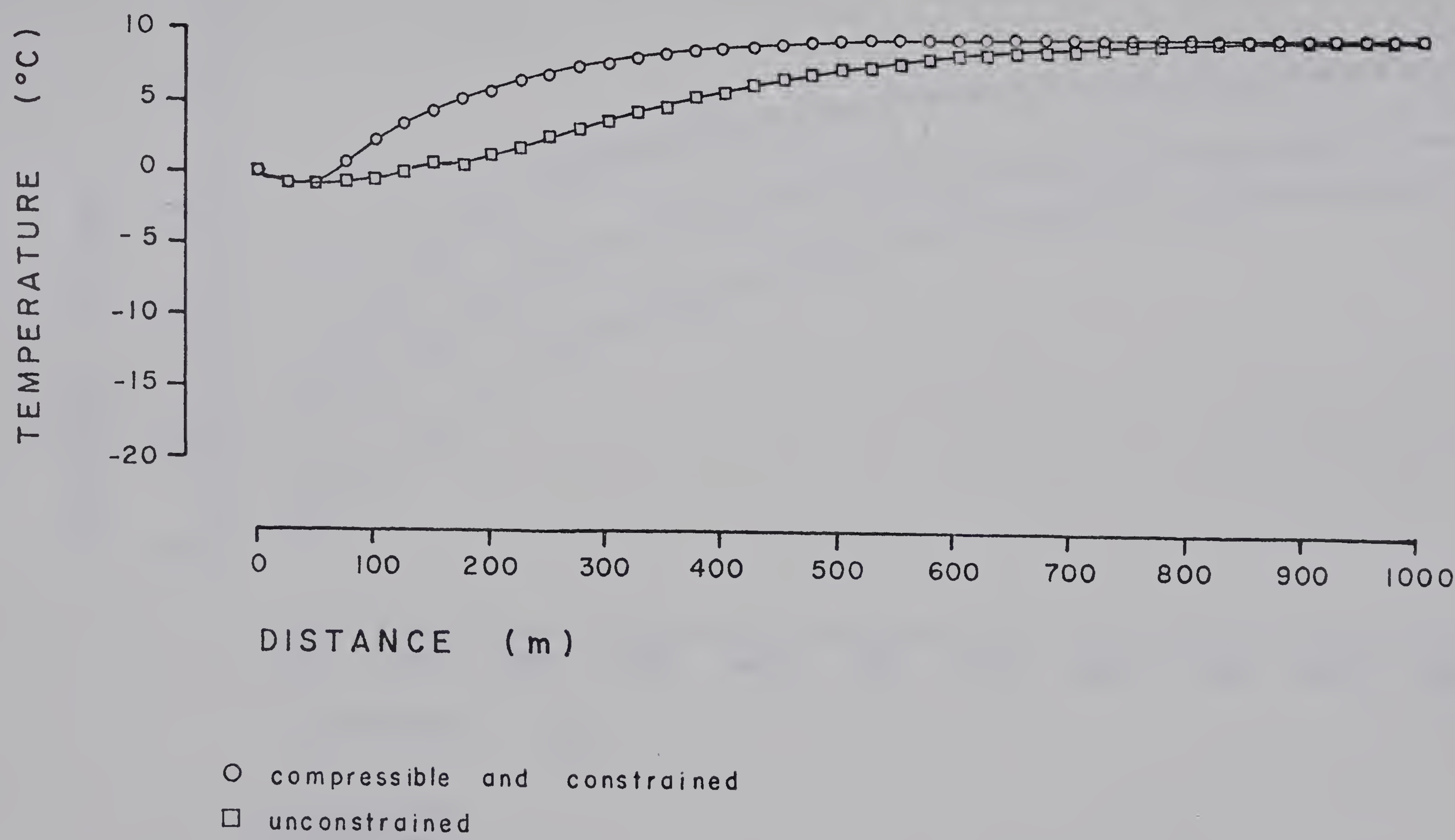


FIGURE 14: Velocity - Case I

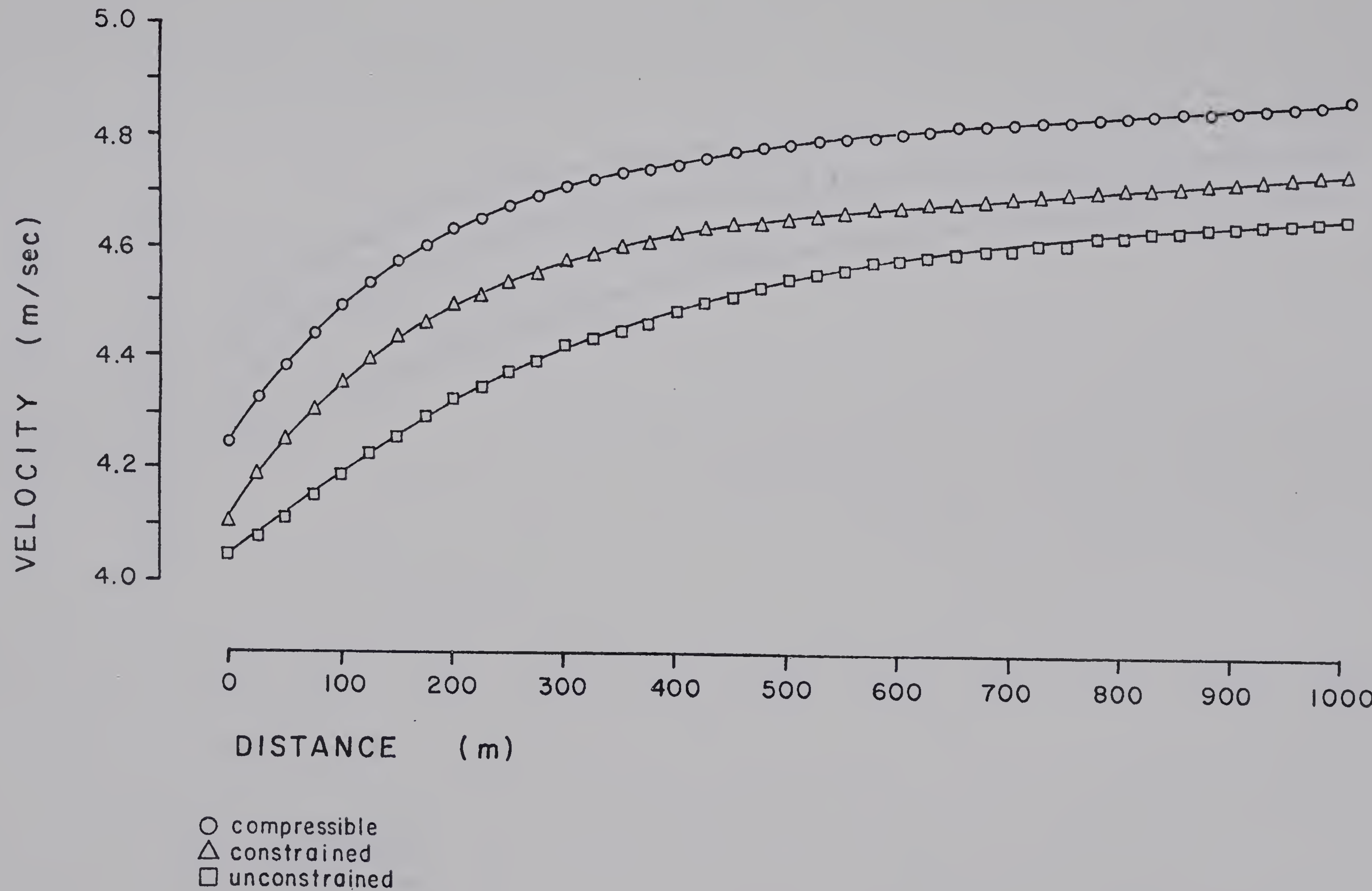


FIGURE 15: Velocity - Case 2

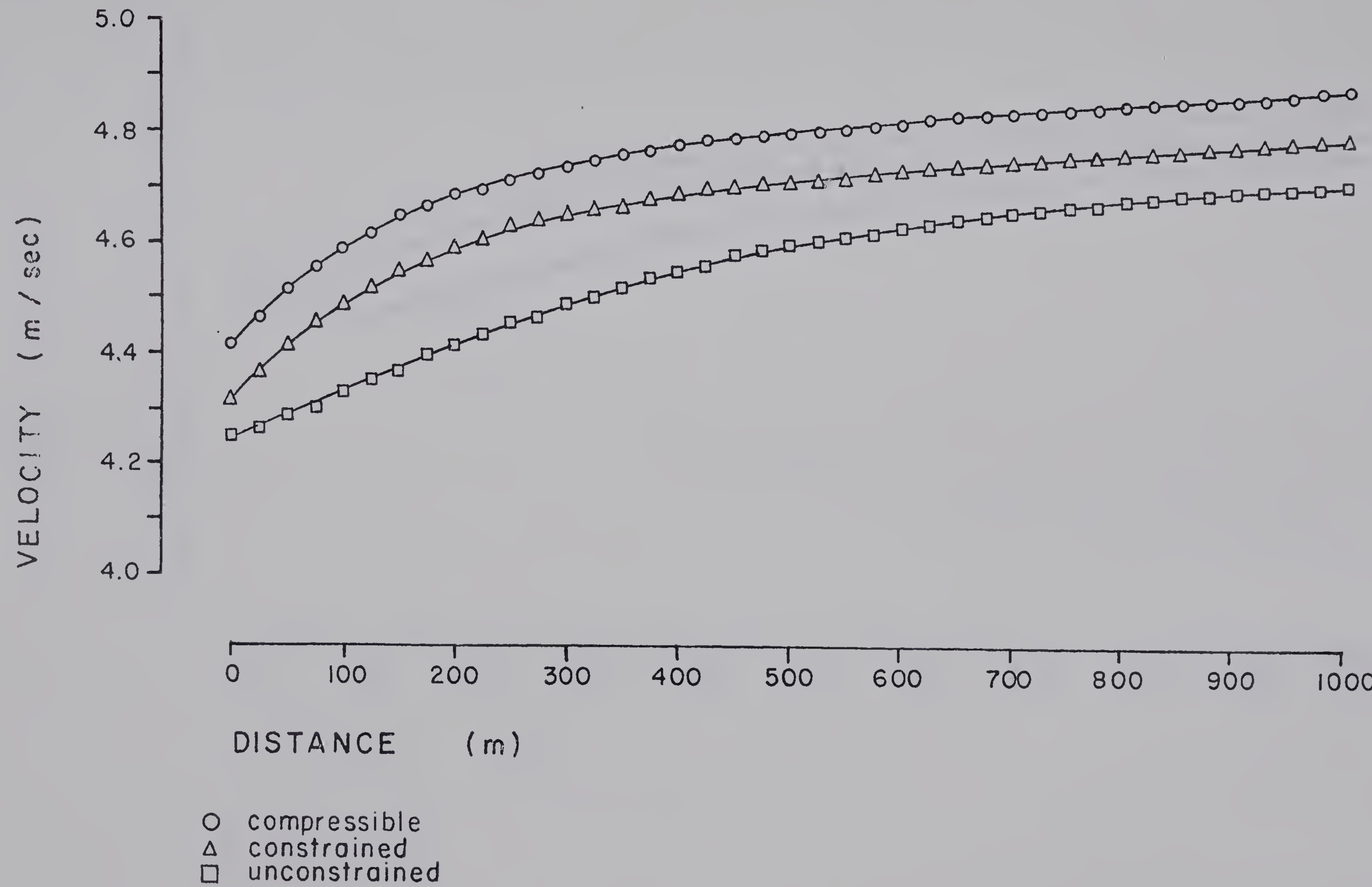


FIGURE 16: VELOCITY - Case 3

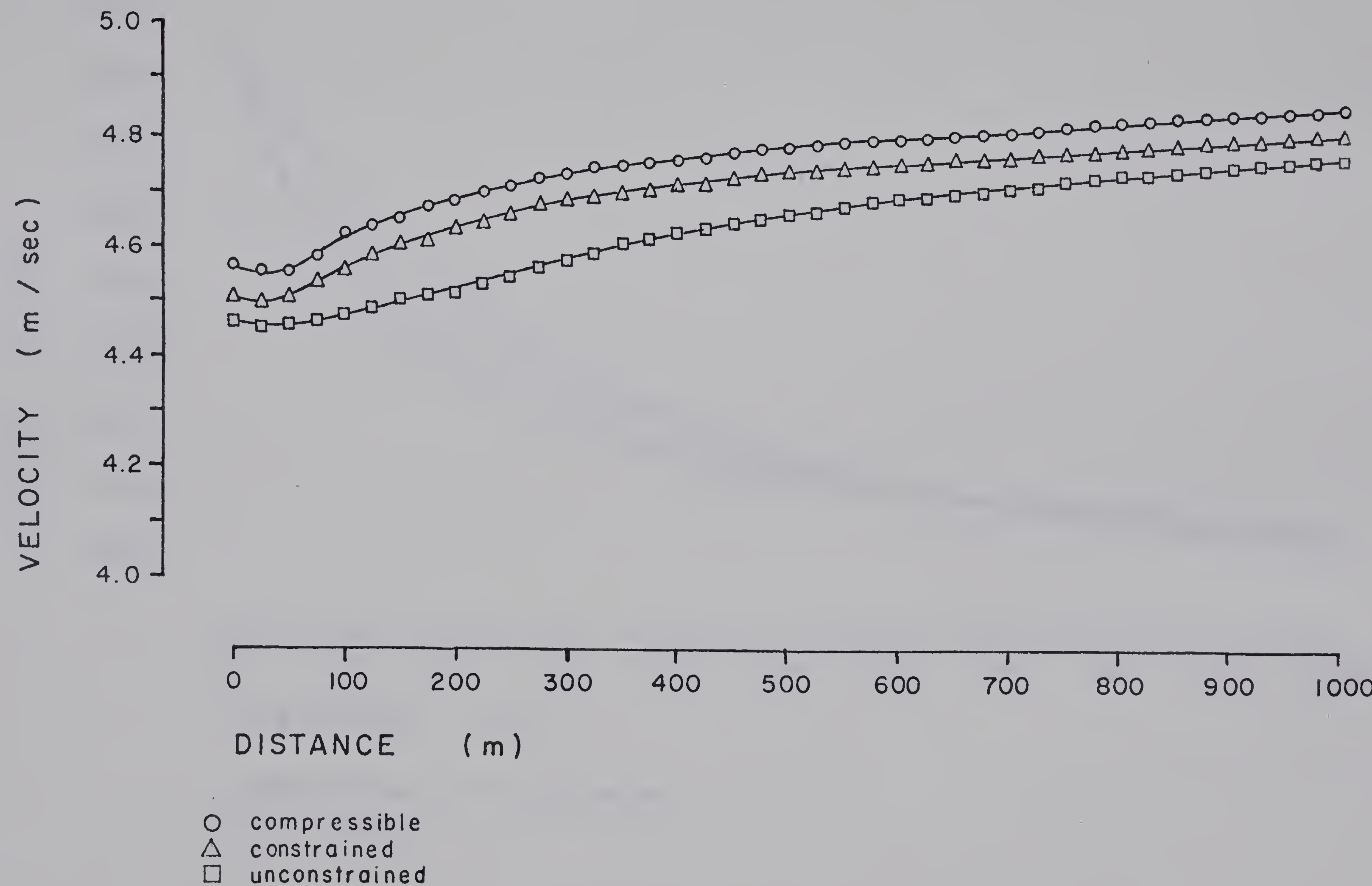


FIGURE 17 : Density - Case I

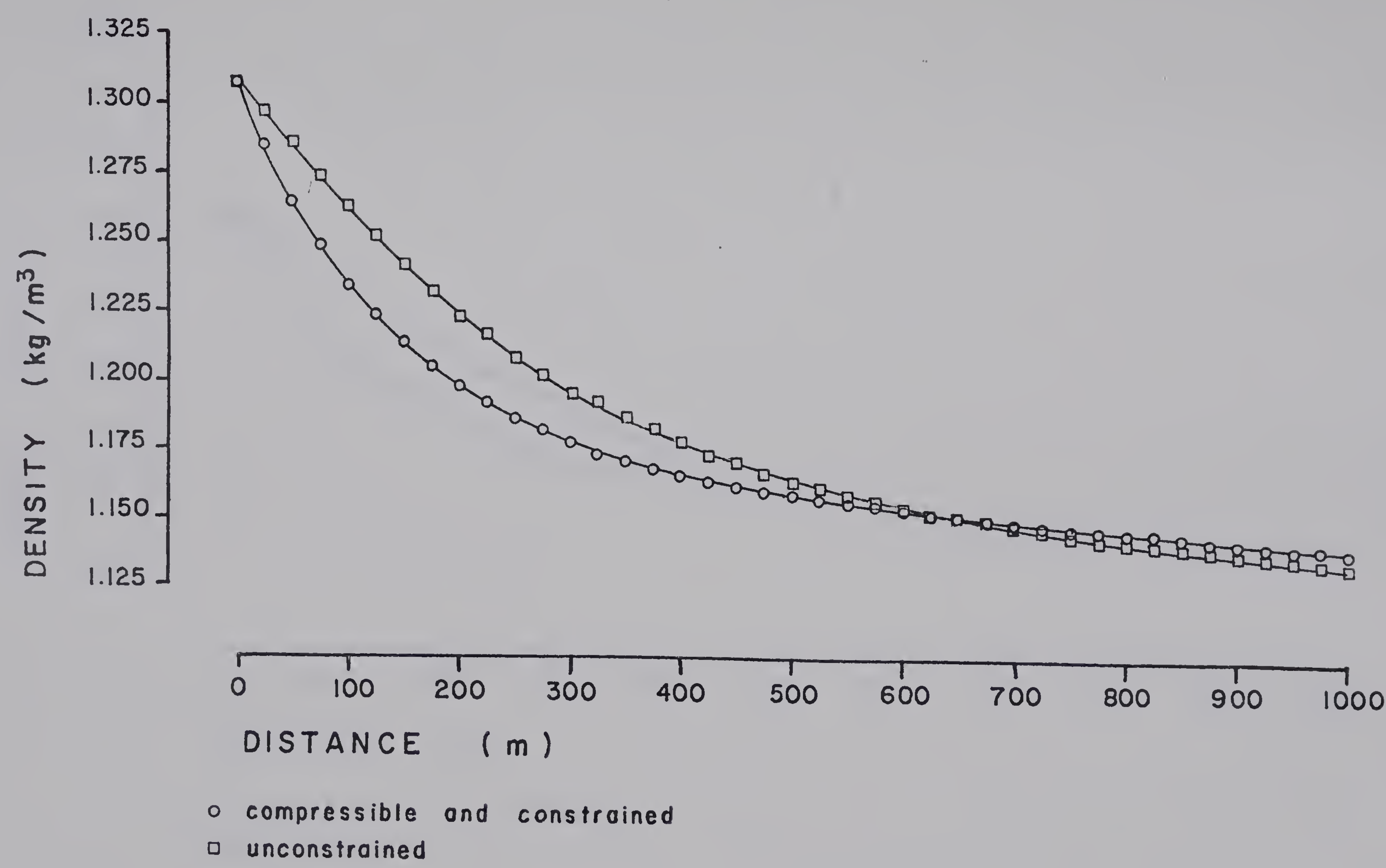


FIGURE 18 : Density - Case 2

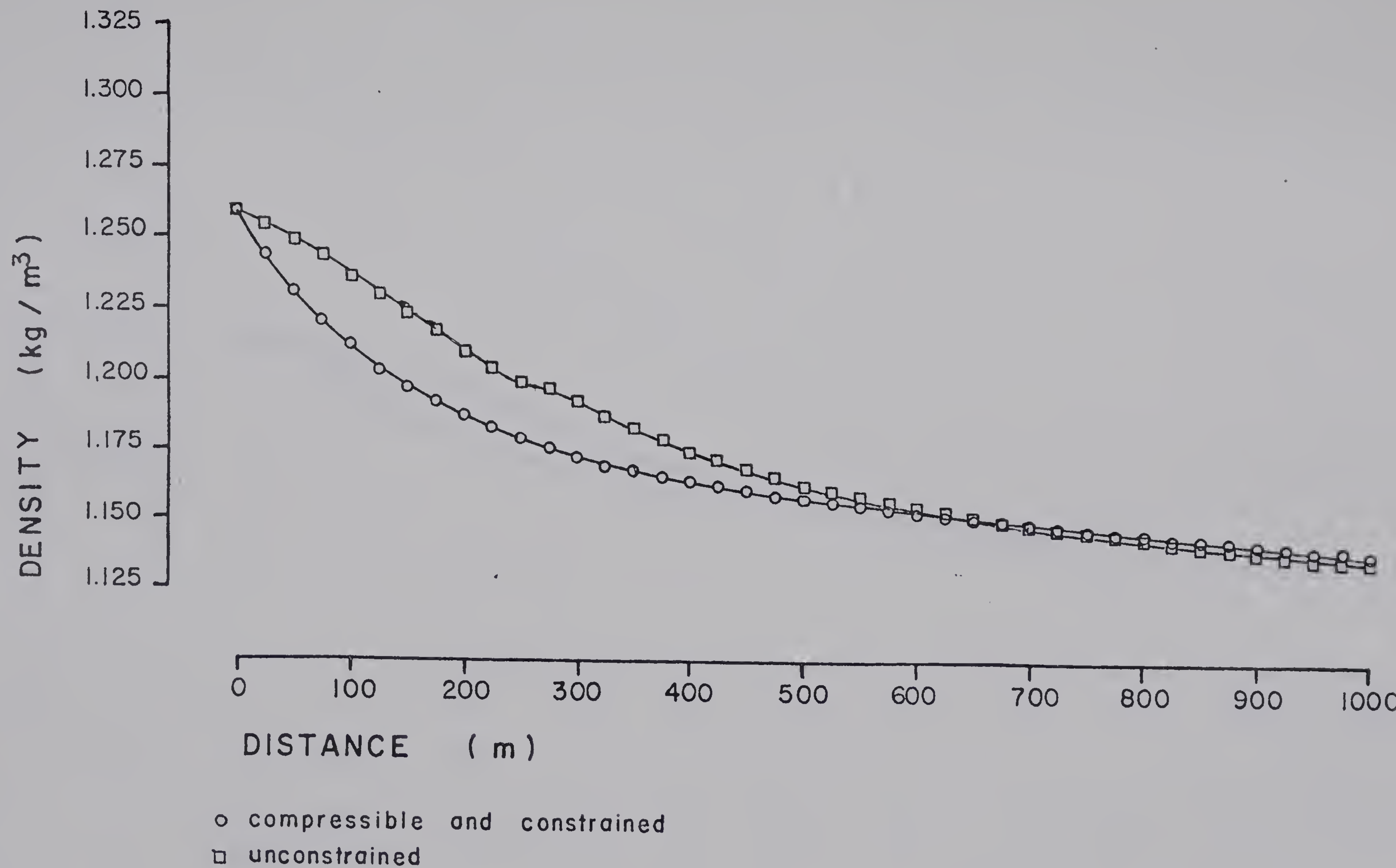


FIGURE 19: Density - Case 3

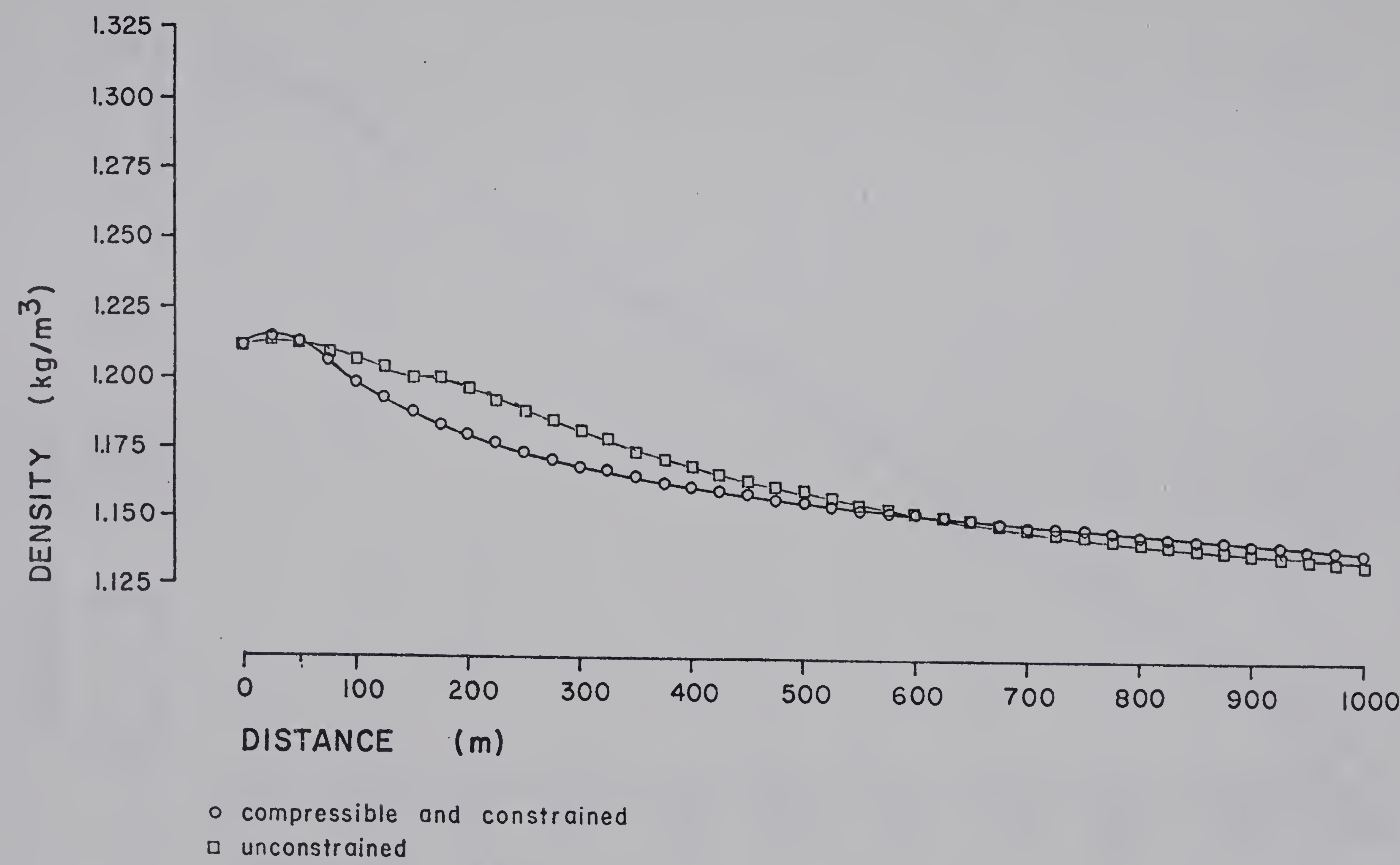
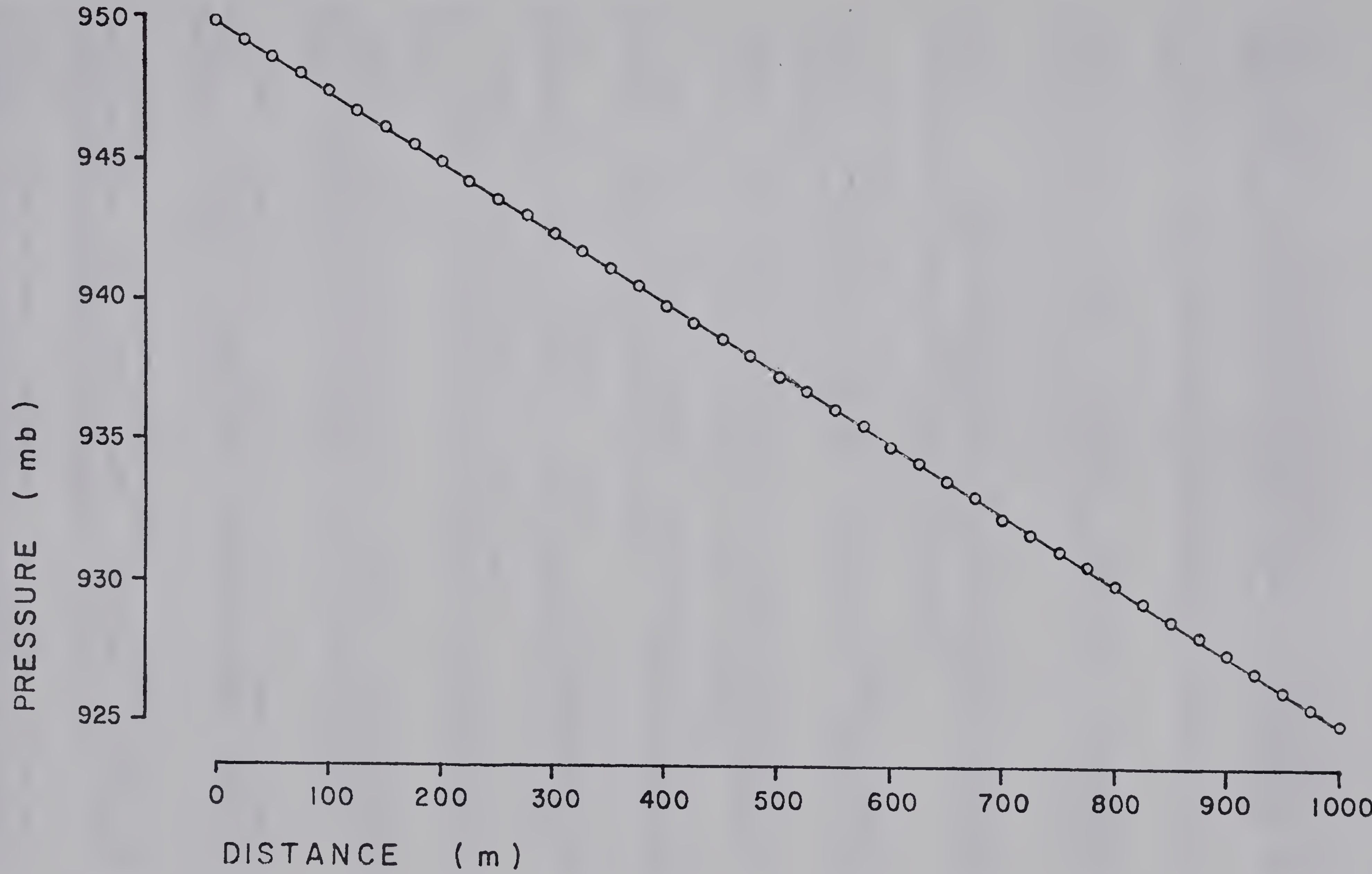


FIGURE 20: Pressure - All Cases and Models



## BIBLIOGRAPHY


Atkinson, J.J. (1886), A Practical Treatise on the Gases Met with in Coal Mines and the General Principles of Ventilation, Longmans, Green, Longmans and Co.

Beard, J.T. (1920), Mine Gases and Ventilation, McGraw-Hill, New York.

Bögli, A. and H.W. Franke (1965), Luminous Darkness, translated by B.M. Charleston and H. Griffin, Kummerly and Frey, Berne.

Brown, M.C. (1972), Karst Hydrology of the Lower Maligne Basin, Cave Studies n.13, Cave Research Associates, Castro Valley, California.

Brown, M.C., D.C. Ford, and T.M.L. Wigley (1969), "Water budget studies in karst aquifers," Journal of Hydrology, 9(1):113-116.

Bunt,H.J. (1960), "The flow of fluids," in Mine Ventilation, edited by A. Roberts, Cleaver-Hume, London.

Carslaw, H.S. and J.C. Jaeger (1959), Conduction of Heat in Solids, Oxford University Press.

Chapman, A.J. and W.F. Walker (1971), Introductory Gas Dynamics, Holt, Rinehart, and Winston, New York.

Cigna, A.A. (1963), "An analytical study of air circulation in caves," International Journal of Speleology - III, p. 41-54.

Cigna,A.A. (1965), "Air circulation in caves," Proceedings of the Fourth International Congress of Speleology, Ljubljana, p. 43-49.

Conn, H.W. (1966), "Barometric wind in Wind and Jewel Caves, South Dakota," National Speleological Society Bulletin, 28(2):55-69.

Cropley, J.B. (1965), "Influence of surface conditions on temperatures in large cave systems," National Speleological Society Bulletin, 27(1):1-10.

Eraso, A. (1964), "Tentative nomogram for cave climate," Problems of the Speleological Research, Proceedings of the International Speleological Conference, Brno, June 29 - July 4, p. 175-185.

Ford, D.C., R.S. Harmon, H.P. Schwarcz, T.M.L. Wigley, and

P. Thompson (1975), "Geohydrologic and thermometric observations in the vicinity of the Columbia Icefield, Alberta-B.C., Canada," paper submitted to the International Symposium on the Thermal Regime of Glaciers and Ice Sheets, Vancouver April, 1975.

Geiger, R. (1966), The Climate Near the Ground, Harvard University Press, Cambridge, Massachusetts.

Hall, C.J. (1953), "Thermodynamics of mine ventilation," Colliery Engineering, 30:66.

Hall, C.J. (1967), Air Flow in Mines , C.J. Hall Associates, Michigan Technological University, Houghton.

Harris, E.J. and D.S. Kingerly (1973), "Ventilation," in Society of Mine Engineers Handbook.

Hartman, H.L. (1961), Mine Ventilation and Air Conditioning, Ronald Press, New York.

Hess, S.L. (1959), Introduction to Theoretical Meteorology, Holt, Rinehart, and Winston, New York.

Hinsley, F.B. (1938), "A new method of evaluating the effects of natural agencies on the ventilation of mines," Transactions of the Institute of Mining Engineers, 97:131-151.

Hinsley, F.B. (1943a), "Air flow in mines: a thermodynamic analysis," The Colliery Guardian, 166(4299):605-608.

Hinsley, F.B. (1943b), "Air flow in mines: a thermodynamic analysis" continued, The Colliery Guardian, 166(4300):635-640.

Hinsley, F.B. (1943c), "Air flow in mines: a thermodynamic analysis" continued, The Colliery Guardian, 166(4301):669-673.

Hinsley, F.B. (1943d), "Air flow in mines: a thermodynamic analysis" continued, The Colliery Guardian, 166(4302):699-701.

Kays, W.M. (1966), Convective Heat and Mass Transfer, McGraw-Hill, New York.

Lange, A. (1954), "Rock temperature distributions underground, part I," Cave Studies 6:21-25.

McElroy, G.E. (1950), "Mine ventilation," in Mining Engineers Handbook, volume I, edited by R. Peele,

John Wiley and Sons, London.

Montoriol Pous, J. (?), "Resultado de las observaciones espeleometeorologicas realizadas durante la exploracion de la Sima de los Esquirols (Macizo de Garraf, Barcelona)."

Moody, L.F. (1946), "Friction factors for pipe flow," Transactions of the American Society of Mechanical Engineers, 13(2):A91.

Moore, G.W., and Brother G. Nicholas (1964), Speleology, the Study of Caves, Heath, Boston.

Myers, J.O. (1962), "Cave physics," in British Caving, edited by C.H.D. Cullingford, Routledge and Kegan Paul, London.

Penman, D. and J.S. Penman (1947), The Principles and Practice of Mine Ventilation, Charles Griffin, London.

Plummer, W.T. (1962), "A note on cave breathing," Speleo Digest - 1962, National Speleological Society.

Plummer, W.T. (1969), "Infrasonic resonances in natural underground cavities," Journal of the Acoustical Society of America, 46(5, part I):1074-1080.

Poulson, T.L. and W.B. White (1969), "The cave environment," Science, 165:971-981.

Schlichting, H. (1968), Boundary Layer Theory, McGraw-Hill, New York.

Schmidt, V.A. (1958), "The ins and outs of cave breathing," Speleo Digest 1958, National Speleological Society p. 2/107.

Shapiro, A.H. (1953), The Dynamics and Thermodynamics of Compressible Fluid Flow, Ronald Press, New York.

Schuster, E.T. and W.B. White (1971), "Seasonal fluctuations in the chemistry of limestone springs: a possible means for characterising carbonate aquifers," Journal of Hydrology, 14:93-128.

Simonys, F. (1913), Hohlen im Dachstein, Deutsche Vereins-Druckerei Graz.

Stǒces, B. (1954), Introduction to Mining, Lange, Maxwell and Springer, London.

Smithsonian Meteorological Tables (1958), Smithsonian

Institution, Washington.

Techo, R., R.R. Tickner, and R.E. James (1965), "An accurate equation for the computation of the friction factor for smooth pipes from the Reynolds number," Journal of Applied Mechanics, 87(3):443.

Thompson, P.A. (1972), Compressible-Fluid Dynamics, McGraw-Hill, New York.

Trimmel, H. (1968), Höhlenkunde, Friedrich Vieweg und Sohn, Braunschweig.

Trombe, F. (1952), Traité de Spéléologie, Payot, Paris.

Vandel, A. (1965), Biospeleolcgy, Pergamon.

Weast, R.C. (1970), Handbook of Chemistry and Physics, The Chemical Rubber Company, Cleveland, Ohio.

Weeks, W.S. (1926), Ventilation of Mines, McGraw-Hill, New Ycrk.

Wigley, T.M.L. (1967), "Non-Steady flow through a porous medium and cave breathing," Journal of Geophysical Research, 72(12): 3199-3205.

Wigley, T.M.L. and M.C. Brown (1969), "Geohydrological implications of cave breathing," Proceedings of the International Congress for Speleology, 1969.

Wigley, T.M.L. and M.C. Brown (1971), "Geophysical applications of heat and mass transfer in turbulent pipe flow," Boundary-Layer Meteorology, 1:300-320.

Wigley, T.M.L. and M.C. Brcwn (in press), "Cave meteorology," in The Science of Speleclogy, edited by T.D. Ford and C.H.D. Cullingford, Routledge and Kegan Paul, London.

Wigley, T.M.L., J.D. Wood, and M. Smith (1966), "Meteorological aspects of Mullamullang Cave," Cave Exploration Group (South Australia) Occasional Paper 4.

Williams, F.T. (1960), "The thermodynamics of mine ventilation," in Mine Ventilation, edited by A. Roberts, Cleaver-Hume Press, London.

## APPENDIX A: CASE STUDIES

### Case 1: Compressible model

-

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -20.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |

-

| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

-

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -19.99 | 950.0 | 1.3073 |
| 25.0 | -15.52 | 949.3 | 1.2836 |
| 50.0 | -11.72 | 948.7 | 1.2642 |
| 75.0 | -8.49 | 948.1 | 1.2479 |
| 100.0 | -5.75 | 947.5 | 1.2342 |
| 125.0 | -3.42 | 946.9 | 1.2227 |
| 150.0 | -1.44 | 946.2 | 1.2130 |
| 175.0 | 0.24 | 945.6 | 1.2047 |
| 200.0 | 1.67 | 945.0 | 1.1976 |
| 225.0 | 2.89 | 944.3 | 1.1915 |
| 250.0 | 3.92 | 943.7 | 1.1862 |
| 275.0 | 4.79 | 943.1 | 1.1817 |
| 300.0 | 5.54 | 942.5 | 1.1777 |
| 325.0 | 6.17 | 941.8 | 1.1742 |
| 350.0 | 6.71 | 941.2 | 1.1712 |
| 375.0 | 7.16 | 940.6 | 1.1685 |
| 400.0 | 7.55 | 939.9 | 1.1661 |
| 425.0 | 7.88 | 939.3 | 1.1639 |
| 450.0 | 8.16 | 938.7 | 1.1620 |
| 475.0 | 8.40 | 938.1 | 1.1602 |
| 500.0 | 8.60 | 937.4 | 1.1586 |
| 525.0 | 8.77 | 936.8 | 1.1571 |
| 550.0 | 8.92 | 936.2 | 1.1557 |
| 575.0 | 9.04 | 935.6 | 1.1545 |
| 600.0 | 9.15 | 934.9 | 1.1532 |
| 625.0 | 9.24 | 934.3 | 1.1521 |
| 650.0 | 9.31 | 933.7 | 1.1510 |
| 675.0 | 9.38 | 933.1 | 1.1500 |
| 700.0 | 9.43 | 932.4 | 1.1490 |
| 725.0 | 9.48 | 931.8 | 1.1480 |
| 750.0 | 9.52 | 931.2 | 1.1471 |
| 775.0 | 9.55 | 930.6 | 1.1462 |
| 800.0 | 9.58 | 930.0 | 1.1453 |
| 825.0 | 9.60 | 929.3 | 1.1445 |

| | | | |
|---|---|---|---|
| 850.0 | 9.62 | 928.7 | 1.1436 |
| 875.0 | 9.64 | 928.1 | 1.1428 |
| 900.0 | 9.66 | 927.5 | 1.1420 |
| 925.0 | 9.67 | 926.9 | 1.1411 |
| 950.0 | 9.68 | 926.2 | 1.1403 |
| 975.0 | 9.68 | 925.6 | 1.1396 |
| 1000.0 | 9.70 | 925.0 | 1.1387 |

| DISTANCE (METRES) | STAGNATION TEMPERATURE (DEG CELSIUS) | STAGNATION PRESSURE (MILLIBARS) | STAGNATION DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -19.98 | 950.1 | 1.3074 |
| 25.0 | -15.51 | 949.5 | 1.2838 |
| 50.0 | -11.71 | 948.9 | 1.2643 |
| 75.0 | -8.48 | 948.2 | 1.2480 |
| 100.0 | -5.74 | 947.6 | 1.2344 |
| 125.0 | -3.41 | 947.0 | 1.2229 |
| 150.0 | -1.43 | 946.4 | 1.2131 |
| 175.0 | 0.25 | 945.7 | 1.2048 |
| 200.0 | 1.68 | 945.1 | 1.1977 |
| 225.0 | 2.90 | 944.5 | 1.1916 |
| 250.0 | 3.93 | 943.8 | 1.1863 |
| 275.0 | 4.80 | 943.2 | 1.1818 |
| 300.0 | 5.55 | 942.6 | 1.1778 |
| 325.0 | 6.18 | 942.0 | 1.1744 |
| 350.0 | 6.72 | 941.3 | 1.1713 |
| 375.0 | 7.17 | 940.7 | 1.1686 |
| 400.0 | 7.56 | 940.1 | 1.1662 |
| 425.0 | 7.89 | 939.5 | 1.1640 |
| 450.0 | 8.17 | 938.8 | 1.1621 |
| 475.0 | 8.41 | 938.2 | 1.1603 |
| 500.0 | 8.61 | 937.6 | 1.1587 |
| 525.0 | 8.78 | 936.9 | 1.1572 |
| 550.0 | 8.93 | 936.3 | 1.1559 |
| 575.0 | 9.05 | 935.7 | 1.1546 |
| 600.0 | 9.16 | 935.1 | 1.1534 |
| 625.0 | 9.25 | 934.5 | 1.1522 |
| 650.0 | 9.32 | 933.8 | 1.1511 |
| 675.0 | 9.39 | 933.2 | 1.1501 |
| 700.0 | 9.44 | 932.6 | 1.1491 |
| 725.0 | 9.49 | 932.0 | 1.1482 |
| 750.0 | 9.53 | 931.3 | 1.1472 |
| 775.0 | 9.56 | 930.7 | 1.1463 |
| 800.0 | 9.59 | 930.1 | 1.1454 |
| 825.0 | 9.61 | 929.5 | 1.1446 |
| 850.0 | 9.64 | 928.9 | 1.1437 |
| 875.0 | 9.65 | 928.2 | 1.1429 |
| 900.0 | 9.67 | 927.6 | 1.1421 |
| 925.0 | 9.68 | 927.0 | 1.1413 |
| 950.0 | 9.69 | 926.4 | 1.1404 |
| 975.0 | 9.69 | 925.7 | 1.1397 |
| 1000.0 | 9.71 | 925.1 | 1.1389 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | MACH NUMBER | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.2492 | 0.013322 | 0.500 |
| 25.0 | 4.3274 | 0.013449 | 1.000 |
| 50.0 | 4.3941 | 0.013556 | 1.000 |
| 75.0 | 4.4514 | 0.013649 | 1.000 |
| 100.0 | 4.5006 | 0.013728 | 1.000 |
| 125.0 | 4.5430 | 0.013797 | 1.000 |
| 150.0 | 4.5795 | 0.013857 | 1.000 |
| 175.0 | 4.6111 | 0.013910 | 1.000 |
| 200.0 | 4.6384 | 0.013956 | 1.000 |
| 225.0 | 4.6621 | 0.013996 | 1.000 |
| 250.0 | 4.6828 | 0.014032 | 1.000 |
| 275.0 | 4.7008 | 0.014063 | 1.000 |
| 300.0 | 4.7167 | 0.014092 | 1.000 |
| 325.0 | 4.7306 | 0.014117 | 1.000 |
| 350.0 | 4.7430 | 0.014140 | 1.000 |
| 375.0 | 4.7539 | 0.014161 | 1.000 |
| 400.0 | 4.7637 | 0.014181 | 1.000 |
| 425.0 | 4.7726 | 0.014199 | 1.000 |
| 450.0 | 4.7805 | 0.014215 | 1.000 |
| 475.0 | 4.7878 | 0.014231 | 1.000 |
| 500.0 | 4.7945 | 0.014245 | 1.000 |
| 525.0 | 4.8006 | 0.014259 | 1.000 |
| 550.0 | 4.8063 | 0.014272 | 1.000 |
| 575.0 | 4.8117 | 0.014285 | 1.000 |
| 600.0 | 4.8167 | 0.014297 | 1.000 |
| 625.0 | 4.8215 | 0.014309 | 1.000 |
| 650.0 | 4.8260 | 0.014321 | 1.000 |
| 675.0 | 4.8303 | 0.014332 | 1.000 |
| 700.0 | 4.8345 | 0.014343 | 1.000 |
| 725.0 | 4.8385 | 0.014354 | 1.000 |
| 750.0 | 4.8425 | 0.014364 | 1.000 |
| 775.0 | 4.8463 | 0.014375 | 1.000 |
| 800.0 | 4.8500 | 0.014385 | 1.000 |
| 825.0 | 4.8537 | 0.014395 | 1.000 |
| 850.0 | 4.8573 | 0.014406 | 1.000 |
| 875.0 | 4.8608 | 0.014416 | 1.000 |
| 900.0 | 4.8644 | 0.014426 | 1.000 |
| 925.0 | 4.8678 | 0.014436 | 1.000 |
| 950.0 | 4.8713 | 0.014446 | 1.000 |
| 975.0 | 4.8745 | 0.014455 | 1.000 |
| 1000.0 | 4.8781 | 0.014465 | 1.000 |

Case 1: Incompressible model - constrained

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -20.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |

NUMBER OF DIVISIONS OF PASSAGE 40

| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -20.00 | 949.9 | 1.3071 |
| 25.0 | -15.51 | 949.2 | 1.2833 |
| 50.0 | -11.70 | 948.6 | 1.2634 |
| 75.0 | -8.46 | 947.9 | 1.2468 |
| 100.0 | -5.71 | 947.2 | 1.2328 |
| 125.0 | -3.38 | 946.6 | 1.2209 |
| 150.0 | -1.40 | 945.9 | 1.2107 |
| 175.0 | 0.28 | 945.3 | 1.2021 |
| 200.0 | 1.71 | 944.6 | 1.1946 |
| 225.0 | 2.92 | 944.0 | 1.1882 |
| 250.0 | 3.95 | 943.3 | 1.1827 |
| 275.0 | 4.82 | 942.7 | 1.1779 |
| 300.0 | 5.57 | 942.0 | 1.1737 |
| 325.0 | 6.20 | 941.4 | 1.1701 |
| 350.0 | 6.73 | 940.7 | 1.1668 |
| 375.0 | 7.19 | 940.1 | 1.1640 |
| 400.0 | 7.57 | 939.4 | 1.1614 |
| 425.0 | 7.90 | 938.8 | 1.1592 |
| 450.0 | 8.18 | 938.1 | 1.1571 |
| 475.0 | 8.41 | 937.5 | 1.1552 |
| 500.0 | 8.61 | 936.8 | 1.1535 |
| 525.0 | 8.78 | 936.2 | 1.1520 |
| 550.0 | 8.93 | 935.6 | 1.1505 |
| 575.0 | 9.05 | 934.9 | 1.1492 |
| 600.0 | 9.16 | 934.3 | 1.1479 |
| 625.0 | 9.24 | 933.6 | 1.1467 |
| 650.0 | 9.32 | 933.0 | 1.1456 |
| 675.0 | 9.38 | 932.3 | 1.1445 |
| 700.0 | 9.44 | 931.7 | 1.1435 |
| 725.0 | 9.48 | 931.0 | 1.1425 |
| 750.0 | 9.52 | 930.4 | 1.1415 |
| 775.0 | 9.55 | 929.7 | 1.1406 |
| 800.0 | 9.58 | 929.1 | 1.1397 |
| 825.0 | 9.61 | 928.5 | 1.1388 |
| 850.0 | 9.63 | 927.8 | 1.1379 |
| 875.0 | 9.64 | 927.2 | 1.1370 |
| 900.0 | 9.66 | 926.5 | 1.1362 |
| 925.0 | 9.67 | 925.9 | 1.1353 |
| 950.0 | 9.68 | 925.2 | 1.1345 |
| 975.0 | 9.68 | 924.6 | 1.1337 |
| 1000.0 | 9.70 | 925.0 | 1.1329 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.1124 | 950.0 | 0.500 |
| 25.0 | 4.1886 | 949.3 | 1.000 |
| 50.0 | 4.2544 | 948.7 | 1.000 |
| 75.0 | 4.3112 | 948.0 | 1.000 |
| 100.0 | 4.3602 | 947.4 | 1.000 |
| 125.0 | 4.4027 | 946.7 | 1.000 |
| 150.0 | 4.4396 | 946.0 | 1.000 |
| 175.0 | 4.4716 | 945.4 | 1.000 |
| 200.0 | 4.4995 | 944.7 | 1.000 |
| 225.0 | 4.5237 | 944.1 | 1.000 |
| 250.0 | 4.5448 | 943.4 | 1.000 |
| 275.0 | 4.5633 | 942.8 | 1.000 |
| 300.0 | 4.5796 | 942.1 | 1.000 |
| 325.0 | 4.5939 | 941.5 | 1.000 |
| 350.0 | 4.6066 | 940.8 | 1.000 |
| 375.0 | 4.6179 | 940.2 | 1.000 |
| 400.0 | 4.6280 | 939.6 | 1.000 |
| 425.0 | 4.6371 | 938.9 | 1.000 |
| 450.0 | 4.6453 | 938.3 | 1.000 |
| 475.0 | 4.6528 | 937.6 | 1.000 |
| 500.0 | 4.6597 | 937.0 | 1.000 |
| 525.0 | 4.6660 | 936.3 | 1.000 |
| 550.0 | 4.6718 | 935.7 | 1.000 |
| 575.0 | 4.6773 | 935.0 | 1.000 |
| 600.0 | 4.6824 | 934.4 | 1.000 |
| 625.0 | 4.6873 | 933.7 | 1.000 |
| 650.0 | 4.6919 | 933.1 | 1.000 |
| 675.0 | 4.6963 | 932.4 | 1.000 |
| 700.0 | 4.7006 | 931.8 | 1.000 |
| 725.0 | 4.7047 | 931.2 | 1.000 |
| 750.0 | 4.7087 | 930.5 | 1.000 |
| 775.0 | 4.7125 | 929.9 | 1.000 |
| 800.0 | 4.7163 | 929.2 | 1.000 |
| 825.0 | 4.7200 | 928.6 | 1.000 |
| 850.0 | 4.7237 | 927.9 | 1.000 |
| 875.0 | 4.7273 | 927.3 | 1.000 |
| 900.0 | 4.7308 | 926.6 | 1.000 |
| 925.0 | 4.7344 | 926.0 | 1.000 |
| 950.0 | 4.7379 | 925.4 | 1.000 |
| 975.0 | 4.7412 | 924.7 | 1.000 |
| 1000.0 | 4.7448 | 925.1 | 1.000 |

Case 1: Inccmpressible model - unconstrained

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -20.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |

NUMBER OF DIVISIONS OF PASSAGE 40

| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -20.00 | 949.9 | 1.3071 |
| 25.0 | -18.18 | 949.2 | 1.2961 |
| 50.0 | -16.23 | 948.6 | 1.2846 |
| 75.0 | -14.24 | 947.9 | 1.2731 |
| 100.0 | -12.25 | 947.2 | 1.2620 |
| 125.0 | -10.31 | 946.6 | 1.2514 |
| 150.0 | -8.46 | 945.9 | 1.2414 |
| 175.0 | -6.71 | 945.3 | 1.2321 |
| 200.0 | -5.07 | 944.6 | 1.2234 |
| 225.0 | -3.56 | 944.0 | 1.2155 |
| 250.0 | -2.16 | 943.3 | 1.2082 |
| 275.0 | -0.88 | 942.7 | 1.2015 |
| 300.0 | 0.29 | 942.0 | 1.1954 |
| 325.0 | 0.70 | 941.4 | 1.1927 |
| 350.0 | 1.71 | 940.7 | 1.1874 |
| 375.0 | 2.61 | 940.1 | 1.1826 |
| 400.0 | 3.43 | 939.5 | 1.1782 |
| 425.0 | 4.17 | 938.8 | 1.1742 |
| 450.0 | 4.82 | 938.2 | 1.1706 |
| 475.0 | 5.41 | 937.5 | 1.1673 |
| 500.0 | 5.93 | 936.9 | 1.1643 |
| 525.0 | 6.39 | 936.2 | 1.1615 |
| 550.0 | 6.80 | 935.6 | 1.1590 |
| 575.0 | 7.16 | 934.9 | 1.1567 |
| 600.0 | 7.48 | 934.3 | 1.1546 |
| 625.0 | 7.76 | 933.7 | 1.1526 |
| 650.0 | 8.01 | 933.0 | 1.1508 |
| 675.0 | 8.23 | 932.4 | 1.1491 |
| 700.0 | 8.42 | 931.7 | 1.1475 |
| 725.0 | 8.59 | 931.1 | 1.1460 |
| 750.0 | 8.74 | 930.5 | 1.1446 |
| 775.0 | 8.87 | 929.8 | 1.1433 |
| 800.0 | 8.98 | 929.2 | 1.1421 |
| 825.0 | 9.08 | 928.5 | 1.1409 |
| 850.0 | 9.17 | 927.9 | 1.1397 |
| 875.0 | 9.24 | 927.3 | 1.1386 |
| 900.0 | 9.31 | 926.6 | 1.1376 |
| 925.0 | 9.37 | 926.0 | 1.1366 |
| 950.0 | 9.42 | 925.3 | 1.1356 |
| 975.0 | 9.46 | 924.7 | 1.1346 |
| 1000.0 | 9.50 | 925.0 | 1.1337 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.0487 | 950.0 | 0.500 |
| 25.0 | 4.0830 | 949.3 | 2.233 |
| 50.0 | 4.1196 | 948.7 | 3.114 |
| 75.0 | 4.1566 | 948.0 | 3.469 |
| 100.0 | 4.1932 | 947.4 | 3.499 |
| 125.0 | 4.2288 | 946.7 | 3.355 |
| 150.0 | 4.2629 | 946.0 | 3.131 |
| 175.0 | 4.2952 | 945.4 | 2.880 |
| 200.0 | 4.3255 | 944.7 | 2.634 |
| 225.0 | 4.3538 | 944.1 | 2.406 |
| 250.0 | 4.3801 | 943.4 | 2.202 |
| 275.0 | 4.4044 | 942.8 | 2.023 |
| 300.0 | 4.4268 | 942.2 | 1.868 |
| 325.0 | 4.4369 | 941.5 | 1.739 |
| 350.0 | 4.4567 | 940.9 | 1.719 |
| 375.0 | 4.4748 | 940.2 | 1.623 |
| 400.0 | 4.4915 | 939.6 | 1.540 |
| 425.0 | 4.5067 | 938.9 | 1.468 |
| 450.0 | 4.5206 | 938.3 | 1.406 |
| 475.0 | 4.5334 | 937.6 | 1.352 |
| 500.0 | 4.5451 | 937.0 | 1.305 |
| 525.0 | 4.5559 | 936.4 | 1.265 |
| 550.0 | 4.5658 | 935.7 | 1.230 |
| 575.0 | 4.5750 | 935.1 | 1.199 |
| 600.0 | 4.5834 | 934.4 | 1.173 |
| 625.0 | 4.5912 | 933.8 | 1.150 |
| 650.0 | 4.5985 | 933.1 | 1.129 |
| 675.0 | 4.6053 | 932.5 | 1.112 |
| 700.0 | 4.6116 | 931.9 | 1.096 |
| 725.0 | 4.6176 | 931.2 | 1.083 |
| 750.0 | 4.6232 | 930.6 | 1.071 |
| 775.0 | 4.6286 | 929.9 | 1.061 |
| 800.0 | 4.6336 | 929.3 | 1.052 |
| 825.0 | 4.6384 | 928.7 | 1.043 |
| 850.0 | 4.6431 | 928.0 | 1.036 |
| 875.0 | 4.6475 | 927.4 | 1.030 |
| 900.0 | 4.6518 | 926.7 | 1.025 |
| 925.0 | 4.6560 | 926.1 | 1.020 |
| 950.0 | 4.6600 | 925.5 | 1.015 |
| 975.0 | 4.6640 | 924.8 | 1.012 |
| 1000.0 | 4.6678 | 925.1 | 1.008 |

Case 2: Compressible model

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -10.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |

NUMBER OF DIVISIONS OF PASSAGE 40

| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -9.99 | 950.0 | 1.2576 |
| 25.0 | -7.02 | 949.4 | 1.2427 |
| 50.0 | -4.50 | 948.8 | 1.2301 |
| 75.0 | -2.36 | 948.1 | 1.2196 |
| 100.0 | -0.54 | 947.5 | 1.2106 |
| 125.0 | 1.01 | 946.9 | 1.2029 |
| 150.0 | 2.32 | 946.2 | 1.1963 |
| 175.0 | 3.44 | 945.6 | 1.1907 |
| 200.0 | 4.39 | 945.0 | 1.1858 |
| 225.0 | 5.19 | 944.4 | 1.1816 |
| 250.0 | 5.87 | 943.7 | 1.1779 |
| 275.0 | 6.46 | 943.1 | 1.1746 |
| 300.0 | 6.95 | 942.5 | 1.1717 |
| 325.0 | 7.37 | 941.8 | 1.1692 |
| 350.0 | 7.73 | 941.2 | 1.1669 |
| 375.0 | 8.03 | 940.6 | 1.1649 |
| 400.0 | 8.29 | 940.0 | 1.1630 |
| 425.0 | 8.51 | 939.3 | 1.1613 |
| 450.0 | 8.69 | 938.7 | 1.1598 |
| 475.0 | 8.85 | 938.1 | 1.1583 |
| 500.0 | 8.98 | 937.4 | 1.1570 |
| 525.0 | 9.10 | 936.8 | 1.1558 |
| 550.0 | 9.19 | 936.2 | 1.1546 |
| 575.0 | 9.28 | 935.6 | 1.1535 |
| 600.0 | 9.35 | 935.0 | 1.1524 |
| 625.0 | 9.40 | 934.3 | 1.1514 |
| 650.0 | 9.46 | 933.7 | 1.1504 |
| 675.0 | 9.50 | 933.1 | 1.1495 |
| 700.0 | 9.53 | 932.5 | 1.1486 |
| 725.0 | 9.57 | 931.8 | 1.1477 |
| 750.0 | 9.59 | 931.2 | 1.1468 |
| 775.0 | 9.61 | 930.6 | 1.1460 |
| 800.0 | 9.63 | 930.0 | 1.1451 |
| 825.0 | 9.65 | 929.3 | 1.1443 |
| 850.0 | 9.66 | 928.7 | 1.1435 |
| 875.0 | 9.67 | 928.1 | 1.1427 |
| 900.0 | 9.67 | 927.5 | 1.1419 |
| 925.0 | 9.69 | 926.9 | 1.1411 |
| 950.0 | 9.69 | 926.2 | 1.1403 |
| 975.0 | 9.71 | 925.6 | 1.1395 |
| 1000.0 | 9.71 | 925.0 | 1.1387 |

| DISTANCE (METRES) | STAGNATION TEMPERATURE (DEG CELSIUS) | STAGNATION PRESSURE (MILLIBARS) | STAGNATION DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -9.98 | 950.1 | 1.2577 |
| 25.0 | -7.01 | 949.5 | 1.2428 |
| 50.0 | -4.49 | 948.9 | 1.2303 |
| 75.0 | -2.35 | 948.3 | 1.2197 |
| 100.0 | -0.53 | 947.6 | 1.2107 |
| 125.0 | 1.02 | 947.0 | 1.2030 |
| 150.0 | 2.33 | 946.4 | 1.1965 |
| 175.0 | 3.45 | 945.7 | 1.1908 |
| 200.0 | 4.40 | 945.1 | 1.1859 |
| 225.0 | 5.20 | 944.5 | 1.1817 |
| 250.0 | 5.89 | 943.9 | 1.1780 |
| 275.0 | 6.47 | 943.2 | 1.1747 |
| 300.0 | 6.96 | 942.6 | 1.1718 |
| 325.0 | 7.38 | 942.0 | 1.1693 |
| 350.0 | 7.74 | 941.3 | 1.1670 |
| 375.0 | 8.04 | 940.7 | 1.1650 |
| 400.0 | 8.30 | 940.1 | 1.1631 |
| 425.0 | 8.52 | 939.5 | 1.1614 |
| 450.0 | 8.70 | 938.8 | 1.1599 |
| 475.0 | 8.86 | 938.2 | 1.1585 |
| 500.0 | 8.99 | 937.6 | 1.1571 |
| 525.0 | 9.11 | 937.0 | 1.1559 |
| 550.0 | 9.20 | 936.3 | 1.1547 |
| 575.0 | 9.29 | 935.7 | 1.1536 |
| 600.0 | 9.36 | 935.1 | 1.1526 |
| 625.0 | 9.42 | 934.5 | 1.1515 |
| 650.0 | 9.47 | 933.8 | 1.1506 |
| 675.0 | 9.51 | 933.2 | 1.1496 |
| 700.0 | 9.55 | 932.6 | 1.1487 |
| 725.0 | 9.58 | 932.0 | 1.1478 |
| 750.0 | 9.60 | 931.3 | 1.1469 |
| 775.0 | 9.63 | 930.7 | 1.1461 |
| 800.0 | 9.64 | 930.1 | 1.1452 |
| 825.0 | 9.66 | 929.5 | 1.1444 |
| 850.0 | 9.67 | 928.9 | 1.1436 |
| 875.0 | 9.69 | 928.2 | 1.1428 |
| 900.0 | 9.69 | 927.6 | 1.1420 |
| 925.0 | 9.70 | 927.0 | 1.1412 |
| 950.0 | 9.70 | 926.4 | 1.1404 |
| 975.0 | 9.72 | 925.8 | 1.1396 |
| 1000.0 | 9.72 | 925.1 | 1.1388 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | MACH NUMBER | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.4200 | 0.013591 | 0.499 |
| 25.0 | 4.4730 | 0.013677 | 1.000 |
| 50.0 | 4.5185 | 0.013751 | 1.000 |
| 75.0 | 4.5577 | 0.013815 | 1.000 |
| 100.0 | 4.5915 | 0.013871 | 1.000 |

| | | | |
|---|---|---|---|
| 125.0 | 4.6208 | 0.013919 | 1.000 |
| 150.0 | 4.6462 | 0.013962 | 1.000 |
| 175.0 | 4.6682 | 0.014000 | 1.000 |
| 200.0 | 4.6875 | 0.014034 | 1.000 |
| 225.0 | 4.7043 | 0.014063 | 1.000 |
| 250.0 | 4.7191 | 0.014090 | 1.000 |
| 275.0 | 4.7322 | 0.014114 | 1.000 |
| 300.0 | 4.7438 | 0.014136 | 1.000 |
| 325.0 | 4.7541 | 0.014157 | 1.000 |
| 350.0 | 4.7634 | 0.014175 | 1.000 |
| 375.0 | 4.7718 | 0.014192 | 1.000 |
| 400.0 | 4.7794 | 0.014208 | 1.000 |
| 425.0 | 4.7863 | 0.014223 | 1.000 |
| 450.0 | 4.7927 | 0.014238 | 1.000 |
| 475.0 | 4.7986 | 0.014251 | 1.000 |
| 500.0 | 4.8041 | 0.014264 | 1.000 |
| 525.0 | 4.8093 | 0.014276 | 1.000 |
| 550.0 | 4.8141 | 0.014288 | 1.000 |
| 575.0 | 4.8188 | 0.014300 | 1.000 |
| 600.0 | 4.8232 | 0.014311 | 1.000 |
| 625.0 | 4.8274 | 0.014323 | 1.000 |
| 650.0 | 4.8315 | 0.014333 | 1.000 |
| 675.0 | 4.8355 | 0.014344 | 1.000 |
| 700.0 | 4.8393 | 0.014355 | 1.000 |
| 725.0 | 4.8431 | 0.014365 | 1.000 |
| 750.0 | 4.8468 | 0.014375 | 1.000 |
| 775.0 | 4.8504 | 0.014385 | 1.000 |
| 800.0 | 4.8540 | 0.014396 | 1.000 |
| 825.0 | 4.8575 | 0.014406 | 1.000 |
| 850.0 | 4.8610 | 0.014416 | 1.000 |
| 875.0 | 4.8645 | 0.014426 | 1.000 |
| 900.0 | 4.8677 | 0.014435 | 1.000 |
| 925.0 | 4.8713 | 0.014445 | 1.000 |
| 950.0 | 4.8746 | 0.014455 | 1.000 |
| 975.0 | 4.8781 | 0.014465 | 1.000 |
| 1000.0 | 4.8813 | 0.014475 | 1.000 |

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Case 2: Incompressible model - constrained

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| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -10.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |

-

| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

-

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -10.00 | 949.9 | 1.2570 |
| 25.0 | -7.02 | 949.2 | 1.2416 |
| 50.0 | -4.50 | 948.6 | 1.2287 |
| 75.0 | -2.35 | 947.9 | 1.2177 |
| 100.0 | -0.53 | 947.3 | 1.2084 |
| 125.0 | 1.02 | 946.6 | 1.2003 |
| 150.0 | 2.34 | 946.0 | 1.1934 |
| 175.0 | 3.45 | 945.3 | 1.1875 |
| 200.0 | 4.40 | 944.7 | 1.1824 |
| 225.0 | 5.21 | 944.1 | 1.1779 |
| 250.0 | 5.89 | 943.4 | 1.1740 |
| 275.0 | 6.47 | 942.8 | 1.1705 |
| 300.0 | 6.96 | 942.1 | 1.1675 |
| 325.0 | 7.38 | 941.5 | 1.1648 |
| 350.0 | 7.74 | 940.8 | 1.1624 |
| 375.0 | 8.04 | 940.2 | 1.1603 |
| 400.0 | 8.30 | 939.6 | 1.1583 |
| 425.0 | 8.51 | 938.9 | 1.1565 |
| 450.0 | 8.70 | 938.3 | 1.1549 |
| 475.0 | 8.86 | 937.6 | 1.1534 |
| 500.0 | 8.99 | 937.0 | 1.1520 |
| 525.0 | 9.10 | 936.4 | 1.1507 |
| 550.0 | 9.20 | 935.7 | 1.1495 |
| 575.0 | 9.28 | 935.1 | 1.1484 |
| 600.0 | 9.35 | 934.4 | 1.1473 |
| 625.0 | 9.41 | 933.8 | 1.1462 |
| 650.0 | 9.46 | 933.2 | 1.1452 |
| 675.0 | 9.50 | 932.5 | 1.1442 |
| 700.0 | 9.54 | 931.9 | 1.1433 |
| 725.0 | 9.57 | 931.2 | 1.1424 |
| 750.0 | 9.59 | 930.6 | 1.1415 |
| 775.0 | 9.62 | 930.0 | 1.1406 |
| 800.0 | 9.64 | 929.3 | 1.1397 |
| 825.0 | 9.65 | 928.7 | 1.1389 |
| 850.0 | 9.66 | 928.0 | 1.1380 |
| 875.0 | 9.68 | 927.4 | 1.1372 |
| 900.0 | 9.68 | 926.8 | 1.1364 |
| 925.0 | 9.69 | 926.1 | 1.1356 |
| 950.0 | 9.69 | 925.5 | 1.1348 |
| 975.0 | 9.71 | 924.8 | 1.1339 |
| 1000.0 | 9.71 | 925.0 | 1.1331 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.3200 | 950.0 | 0.500 |
| 25.0 | 4.3736 | 949.3 | 1.000 |
| 50.0 | 4.4195 | 948.7 | 1.000 |
| 75.0 | 4.4593 | 948.0 | 1.000 |
| 100.0 | 4.4939 | 947.4 | 1.000 |

| | | | |
|---|---|---|---|
| 125.0 | 4.5240 | 946.8 | 1.000 |
| 150.0 | 4.5501 | 946.1 | 1.000 |
| 175.0 | 4.5728 | 945.5 | 1.000 |
| 200.0 | 4.5927 | 944.8 | 1.000 |
| 225.0 | 4.6101 | 944.2 | 1.000 |
| 250.0 | 4.6255 | 943.5 | 1.000 |
| 275.0 | 4.6391 | 942.9 | 1.000 |
| 300.0 | 4.6511 | 942.3 | 1.000 |
| 325.0 | 4.6619 | 941.6 | 1.000 |
| 350.0 | 4.6715 | 941.0 | 1.000 |
| 375.0 | 4.6802 | 940.3 | 1.000 |
| 400.0 | 4.6880 | 939.7 | 1.000 |
| 425.0 | 4.6952 | 939.0 | 1.000 |
| 450.0 | 4.7018 | 938.4 | 1.000 |
| 475.0 | 4.7079 | 937.8 | 1.000 |
| 500.0 | 4.7136 | 937.1 | 1.000 |
| 525.0 | 4.7189 | 936.5 | 1.000 |
| 550.0 | 4.7239 | 935.8 | 1.000 |
| 575.0 | 4.7286 | 935.2 | 1.000 |
| 600.0 | 4.7332 | 934.6 | 1.000 |
| 625.0 | 4.7375 | 933.9 | 1.000 |
| 650.0 | 4.7417 | 933.3 | 1.000 |
| 675.0 | 4.7457 | 932.6 | 1.000 |
| 700.0 | 4.7496 | 932.0 | 1.000 |
| 725.0 | 4.7535 | 931.4 | 1.000 |
| 750.0 | 4.7572 | 930.7 | 1.000 |
| 775.0 | 4.7609 | 930.1 | 1.000 |
| 800.0 | 4.7645 | 929.4 | 1.000 |
| 825.0 | 4.7681 | 928.8 | 1.000 |
| 850.0 | 4.7716 | 928.2 | 1.000 |
| 875.0 | 4.7751 | 927.5 | 1.000 |
| 900.0 | 4.7784 | 926.9 | 1.000 |
| 925.0 | 4.7820 | 926.3 | 1.000 |
| 950.0 | 4.7854 | 925.6 | 1.000 |
| 975.0 | 4.7889 | 925.0 | 1.000 |
| 1000.0 | 4.7922 | 925.1 | 1.000 |

Case 2: Incompressible model - unconstrained

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -10.000 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | -10.00 | 949.9 | 1.2570 |
| 25.0 | -9.52 | 949.2 | 1.2532 |
| 50.0 | -8.73 | 948.6 | 1.2479 |
| 75.0 | -7.74 | 947.9 | 1.2418 |
| 100.0 | -6.63 | 947.3 | 1.2353 |
| 125.0 | -5.45 | 946.6 | 1.2286 |
| 150.0 | -4.26 | 946.0 | 1.2219 |
| 175.0 | -3.08 | 945.4 | 1.2155 |
| 200.0 | -1.94 | 944.7 | 1.2093 |
| 225.0 | -0.85 | 944.1 | 1.2034 |
| 250.0 | 0.18 | 943.4 | 1.1979 |
| 275.0 | 0.43 | 942.8 | 1.1958 |
| 300.0 | 1.37 | 942.1 | 1.1908 |
| 325.0 | 2.24 | 941.5 | 1.1861 |
| 350.0 | 3.04 | 940.9 | 1.1818 |
| 375.0 | 3.76 | 940.2 | 1.1778 |
| 400.0 | 4.42 | 939.6 | 1.1742 |
| 425.0 | 5.02 | 939.0 | 1.1708 |
| 450.0 | 5.56 | 938.3 | 1.1677 |
| 475.0 | 6.04 | 937.7 | 1.1648 |
| 500.0 | 6.47 | 937.0 | 1.1622 |
| 525.0 | 6.86 | 936.4 | 1.1598 |
| 550.0 | 7.21 | 935.8 | 1.1576 |
| 575.0 | 7.51 | 935.1 | 1.1555 |
| 600.0 | 7.78 | 934.5 | 1.1536 |
| 625.0 | 8.02 | 933.9 | 1.1518 |
| 650.0 | 8.24 | 933.2 | 1.1501 |
| 675.0 | 8.42 | 932.6 | 1.1486 |
| 700.0 | 8.59 | 932.0 | 1.1471 |
| 725.0 | 8.73 | 931.3 | 1.1457 |
| 750.0 | 8.86 | 930.7 | 1.1444 |
| 775.0 | 8.97 | 930.0 | 1.1432 |
| 800.0 | 9.07 | 929.4 | 1.1420 |
| 825.0 | 9.16 | 928.8 | 1.1409 |
| 850.0 | 9.23 | 928.1 | 1.1398 |
| 875.0 | 9.30 | 927.5 | 1.1387 |
| 900.0 | 9.36 | 926.9 | 1.1377 |
| 925.0 | 9.41 | 926.2 | 1.1367 |
| 950.0 | 9.45 | 925.6 | 1.1358 |
| 975.0 | 9.49 | 925.0 | 1.1348 |
| 1000.0 | 9.52 | 925.0 | 1.1339 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.2506 | 950.0 | 0.500 |
| 25.0 | 4.2636 | 949.4 | 1.141 |
| 50.0 | 4.2817 | 948.7 | 1.616 |
| 75.0 | 4.3027 | 948.1 | 1.920 |
| 100.0 | 4.3254 | 947.4 | 2.081 |

| | | | |
|---|---|---|---|
| 125.0 | 4.3489 | 946.8 | 2.137 |
| 150.0 | 4.3726 | 946.1 | 2.120 |
| 175.0 | 4.3958 | 945.5 | 2.058 |
| 200.0 | 4.4183 | 944.8 | 1.973 |
| 225.0 | 4.4399 | 944.2 | 1.877 |
| 250.0 | 4.4603 | 943.5 | 1.779 |
| 275.0 | 4.4681 | 942.9 | 1.688 |
| 300.0 | 4.4869 | 942.3 | 1.699 |
| 325.0 | 4.5046 | 941.6 | 1.621 |
| 350.0 | 4.5210 | 941.0 | 1.549 |
| 375.0 | 4.5363 | 940.3 | 1.484 |
| 400.0 | 4.5505 | 939.7 | 1.426 |
| 425.0 | 4.5636 | 939.1 | 1.374 |
| 450.0 | 4.5757 | 938.4 | 1.328 |
| 475.0 | 4.5869 | 937.8 | 1.288 |
| 500.0 | 4.5973 | 937.2 | 1.252 |
| 525.0 | 4.6069 | 936.5 | 1.220 |
| 550.0 | 4.6158 | 935.9 | 1.193 |
| 575.0 | 4.6240 | 935.3 | 1.168 |
| 600.0 | 4.6317 | 934.6 | 1.146 |
| 625.0 | 4.6389 | 934.0 | 1.128 |
| 650.0 | 4.6456 | 933.3 | 1.111 |
| 675.0 | 4.6519 | 932.7 | 1.096 |
| 700.0 | 4.6578 | 932.1 | 1.083 |
| 725.0 | 4.6635 | 931.4 | 1.072 |
| 750.0 | 4.6688 | 930.8 | 1.062 |
| 775.0 | 4.6738 | 930.2 | 1.053 |
| 800.0 | 4.6787 | 929.5 | 1.045 |
| 825.0 | 4.6833 | 928.9 | 1.038 |
| 850.0 | 4.6878 | 928.3 | 1.031 |
| 875.0 | 4.6921 | 927.6 | 1.026 |
| 900.0 | 4.6963 | 927.0 | 1.021 |
| 925.0 | 4.7003 | 926.4 | 1.017 |
| 950.0 | 4.7043 | 925.7 | 1.013 |
| 975.0 | 4.7081 | 925.1 | 1.010 |
| 1000.0 | 4.7119 | 925.1 | 1.006 |

Case 3: Compressible model

| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -.0 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | 0.01 | 950.0 | 1.2114 |
| 25.0 | -0.89 | 949.4 | 1.2146 |
| 50.0 | -0.84 | 948.7 | 1.2134 |
| 75.0 | 0.75 | 948.1 | 1.2056 |
| 100.0 | 2.11 | 947.5 | 1.1988 |
| 125.0 | 3.26 | 946.8 | 1.1930 |
| 150.0 | 4.23 | 946.2 | 1.1880 |
| 175.0 | 5.06 | 945.6 | 1.1836 |
| 200.0 | 5.76 | 944.9 | 1.1798 |
| 225.0 | 6.36 | 944.3 | 1.1765 |
| 250.0 | 6.87 | 943.7 | 1.1736 |
| 275.0 | 7.30 | 943.1 | 1.1710 |
| 300.0 | 7.67 | 942.4 | 1.1687 |
| 325.0 | 7.98 | 941.8 | 1.1666 |
| 350.0 | 8.25 | 941.2 | 1.1647 |
| 375.0 | 8.47 | 940.6 | 1.1630 |
| 400.0 | 8.66 | 939.9 | 1.1614 |
| 425.0 | 8.82 | 939.3 | 1.1600 |
| 450.0 | 8.96 | 938.7 | 1.1586 |
| 475.0 | 9.08 | 938.1 | 1.1574 |
| 500.0 | 9.18 | 937.4 | 1.1562 |
| 525.0 | 9.26 | 936.8 | 1.1551 |
| 550.0 | 9.33 | 936.2 | 1.1540 |
| 575.0 | 9.40 | 935.6 | 1.1530 |
| 600.0 | 9.45 | 934.9 | 1.1520 |
| 625.0 | 9.49 | 934.3 | 1.1510 |
| 650.0 | 9.53 | 933.7 | 1.1501 |
| 675.0 | 9.56 | 933.1 | 1.1492 |
| 700.0 | 9.59 | 932.4 | 1.1484 |
| 725.0 | 9.61 | 931.8 | 1.1475 |
| 750.0 | 9.63 | 931.2 | 1.1467 |
| 775.0 | 9.65 | 930.6 | 1.1458 |
| 800.0 | 9.66 | 930.0 | 1.1450 |
| 825.0 | 9.67 | 929.3 | 1.1442 |
| 850.0 | 9.68 | 928.7 | 1.1434 |
| 875.0 | 9.68 | 928.1 | 1.1426 |
| 900.0 | 9.70 | 927.5 | 1.1418 |
| 925.0 | 9.70 | 926.9 | 1.1410 |
| 950.0 | 9.71 | 926.2 | 1.1402 |
| 975.0 | 9.71 | 925.6 | 1.1395 |
| 1000.0 | 9.71 | 925.0 | 1.1387 |

| DISTANCE (METRES) | STAGNATION TEMPERATURE (DEG CELSIUS) | STAGNATION PRESSURE (MILLIBARS) | STAGNATION DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | 0.02 | 950.1 | 1.2115 |
| 25.0 | -0.88 | 949.5 | 1.2147 |
| 50.0 | -0.83 | 948.9 | 1.2136 |
| 75.0 | 0.76 | 948.2 | 1.2057 |
| 100.0 | 2.12 | 947.6 | 1.1989 |

| | | | |
|---|---|---|---|
| 125.0 | 3.27 | 947.0 | 1.1931 |
| 150.0 | 4.24 | 946.3 | 1.1881 |
| 175.0 | 5.07 | 945.7 | 1.1838 |
| 200.0 | 5.78 | 945.1 | 1.1800 |
| 225.0 | 6.37 | 944.4 | 1.1766 |
| 250.0 | 6.88 | 943.8 | 1.1737 |
| 275.0 | 7.31 | 943.2 | 1.1711 |
| 300.0 | 7.68 | 942.6 | 1.1688 |
| 325.0 | 7.99 | 941.9 | 1.1667 |
| 350.0 | 8.26 | 941.3 | 1.1648 |
| 375.0 | 8.48 | 940.7 | 1.1631 |
| 400.0 | 8.67 | 940.1 | 1.1615 |
| 425.0 | 8.84 | 939.4 | 1.1601 |
| 450.0 | 8.97 | 938.8 | 1.1587 |
| 475.0 | 9.09 | 938.2 | 1.1575 |
| 500.0 | 9.19 | 937.6 | 1.1563 |
| 525.0 | 9.27 | 936.9 | 1.1552 |
| 550.0 | 9.35 | 936.3 | 1.1541 |
| 575.0 | 9.41 | 935.7 | 1.1531 |
| 600.0 | 9.46 | 935.1 | 1.1521 |
| 625.0 | 9.50 | 934.4 | 1.1512 |
| 650.0 | 9.54 | 933.8 | 1.1502 |
| 675.0 | 9.57 | 933.2 | 1.1493 |
| 700.0 | 9.60 | 932.6 | 1.1485 |
| 725.0 | 9.62 | 932.0 | 1.1476 |
| 750.0 | 9.64 | 931.3 | 1.1468 |
| 775.0 | 9.66 | 930.7 | 1.1459 |
| 800.0 | 9.67 | 930.1 | 1.1451 |
| 825.0 | 9.68 | 929.5 | 1.1443 |
| 850.0 | 9.69 | 928.9 | 1.1435 |
| 875.0 | 9.69 | 928.2 | 1.1427 |
| 900.0 | 9.71 | 927.6 | 1.1419 |
| 925.0 | 9.71 | 927.0 | 1.1411 |
| 950.0 | 9.72 | 926.4 | 1.1403 |
| 975.0 | 9.72 | 925.8 | 1.1396 |
| 1000.0 | 9.72 | 925.1 | 1.1388 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | MACH NUMBER | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.5701 | 0.013793 | 0.500 |
| 25.0 | 4.5583 | 0.013779 | 0.727 |
| 50.0 | 4.5626 | 0.013791 | 0.993 |
| 75.0 | 4.5923 | 0.013840 | 1.000 |
| 100.0 | 4.6182 | 0.013884 | 1.000 |
| 125.0 | 4.6407 | 0.013922 | 1.000 |
| 150.0 | 4.6603 | 0.013956 | 1.000 |
| 175.0 | 4.6774 | 0.013986 | 1.000 |
| 200.0 | 4.6925 | 0.014013 | 1.000 |
| 225.0 | 4.7057 | 0.014038 | 1.000 |
| 250.0 | 4.7175 | 0.014060 | 1.000 |
| 275.0 | 4.7280 | 0.014080 | 1.000 |
| 300.0 | 4.7374 | 0.014099 | 1.000 |
| 325.0 | 4.7458 | 0.014116 | 1.000 |

| | | | |
|---|---|---|---|
| 350.0 | 4.7535 | 0.014133 | 1.000 |
| 375.0 | 4.7605 | 0.014148 | 1.000 |
| 400.0 | 4.7669 | 0.014162 | 1.000 |
| 425.0 | 4.7729 | 0.014175 | 1.000 |
| 450.0 | 4.7784 | 0.014188 | 1.000 |
| 475.0 | 4.7836 | 0.014201 | 1.000 |
| 500.0 | 4.7885 | 0.014213 | 1.000 |
| 525.0 | 4.7931 | 0.014224 | 1.000 |
| 550.0 | 4.7976 | 0.014236 | 1.000 |
| 575.0 | 4.8018 | 0.014247 | 1.000 |
| 600.0 | 4.8059 | 0.014258 | 1.000 |
| 625.0 | 4.8099 | 0.014268 | 1.000 |
| 650.0 | 4.8137 | 0.014279 | 1.000 |
| 675.0 | 4.8175 | 0.014289 | 1.000 |
| 700.0 | 4.8212 | 0.014299 | 1.000 |
| 725.0 | 4.8248 | 0.014309 | 1.000 |
| 750.0 | 4.8283 | 0.014319 | 1.000 |
| 775.0 | 4.8318 | 0.014329 | 1.000 |
| 800.0 | 4.8353 | 0.014339 | 1.000 |
| 825.0 | 4.8387 | 0.014349 | 1.000 |
| 850.0 | 4.8421 | 0.014359 | 1.000 |
| 875.0 | 4.8454 | 0.014369 | 1.000 |
| 900.0 | 4.8489 | 0.014379 | 1.000 |
| 925.0 | 4.8521 | 0.014388 | 1.000 |
| 950.0 | 4.8556 | 0.014398 | 1.000 |
| 975.0 | 4.8588 | 0.014408 | 1.000 |
| 1000.0 | 4.8621 | 0.014417 | 1.000 |

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Case 3: Incompressible model - constrained

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| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -.0 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | 0.0 | 949.9 | 1.2101 |
| 25.0 | -0.90 | 949.2 | 1.2128 |
| 50.0 | -0.84 | 948.6 | 1.2112 |
| 75.0 | 0.75 | 948.0 | 1.2033 |
| 100.0 | 2.11 | 947.3 | 1.1962 |
| 125.0 | 3.26 | 946.7 | 1.1901 |

| | | | |
|---|---|---|---|
| 150.0 | 4.23 | 946.0 | 1.1848 |
| 175.0 | 5.06 | 945.4 | 1.1802 |
| 200.0 | 5.77 | 944.8 | 1.1762 |
| 225.0 | 6.37 | 944.1 | 1.1727 |
| 250.0 | 6.88 | 943.5 | 1.1696 |
| 275.0 | 7.31 | 942.9 | 1.1669 |
| 300.0 | 7.67 | 942.2 | 1.1644 |
| 325.0 | 7.99 | 941.6 | 1.1622 |
| 350.0 | 8.25 | 941.0 | 1.1603 |
| 375.0 | 8.47 | 940.3 | 1.1585 |
| 400.0 | 8.67 | 939.7 | 1.1568 |
| 425.0 | 8.83 | 939.1 | 1.1553 |
| 450.0 | 8.97 | 938.4 | 1.1539 |
| 475.0 | 9.08 | 937.8 | 1.1526 |
| 500.0 | 9.18 | 937.2 | 1.1514 |
| 525.0 | 9.27 | 936.5 | 1.1502 |
| 550.0 | 9.34 | 935.9 | 1.1491 |
| 575.0 | 9.40 | 935.3 | 1.1481 |
| 600.0 | 9.45 | 934.6 | 1.1471 |
| 625.0 | 9.49 | 934.0 | 1.1461 |
| 650.0 | 9.53 | 933.4 | 1.1452 |
| 675.0 | 9.56 | 932.7 | 1.1442 |
| 700.0 | 9.59 | 932.1 | 1.1434 |
| 725.0 | 9.61 | 931.5 | 1.1425 |
| 750.0 | 9.63 | 930.8 | 1.1416 |
| 775.0 | 9.65 | 930.2 | 1.1408 |
| 800.0 | 9.66 | 929.6 | 1.1399 |
| 825.0 | 9.67 | 929.0 | 1.1391 |
| 850.0 | 9.68 | 928.3 | 1.1383 |
| 875.0 | 9.68 | 927.7 | 1.1375 |
| 900.0 | 9.70 | 927.1 | 1.1367 |
| 925.0 | 9.70 | 926.4 | 1.1359 |
| 950.0 | 9.71 | 925.8 | 1.1351 |
| 975.0 | 9.71 | 925.2 | 1.1343 |
| 1000.0 | 9.71 | 925.0 | 1.1335 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.5100 | 950.0 | 0.500 |
| 25.0 | 4.5000 | 949.4 | 0.728 |
| 50.0 | 4.5062 | 948.7 | 0.995 |
| 75.0 | 4.5357 | 948.1 | 1.000 |
| 100.0 | 4.5626 | 947.4 | 1.000 |
| 125.0 | 4.5860 | 946.8 | 1.000 |
| 150.0 | 4.6064 | 946.2 | 1.000 |
| 175.0 | 4.6243 | 945.5 | 1.000 |
| 200.0 | 4.6400 | 944.9 | 1.000 |
| 225.0 | 4.6539 | 944.3 | 1.000 |
| 250.0 | 4.6662 | 943.6 | 1.000 |
| 275.0 | 4.6772 | 943.0 | 1.000 |
| 300.0 | 4.6870 | 942.4 | 1.000 |
| 325.0 | 4.6959 | 941.7 | 1.000 |

| | | | |
|---|---|---|---|
| 350.0 | 4.7039 | 941.1 | 1.000 |
| 375.0 | 4.7111 | 940.5 | 1.000 |
| 400.0 | 4.7178 | 939.8 | 1.000 |
| 425.0 | 4.7240 | 939.2 | 1.000 |
| 450.0 | 4.7297 | 938.6 | 1.000 |
| 475.0 | 4.7351 | 937.9 | 1.000 |
| 500.0 | 4.7401 | 937.3 | 1.000 |
| 525.0 | 4.7449 | 936.7 | 1.000 |
| 550.0 | 4.7494 | 936.0 | 1.000 |
| 575.0 | 4.7537 | 935.4 | 1.000 |
| 600.0 | 4.7579 | 934.8 | 1.000 |
| 625.0 | 4.7620 | 934.1 | 1.000 |
| 650.0 | 4.7659 | 933.5 | 1.000 |
| 675.0 | 4.7697 | 932.9 | 1.000 |
| 700.0 | 4.7734 | 932.2 | 1.000 |
| 725.0 | 4.7771 | 931.6 | 1.000 |
| 750.0 | 4.7807 | 931.0 | 1.000 |
| 775.0 | 4.7842 | 930.3 | 1.000 |
| 800.0 | 4.7877 | 929.7 | 1.000 |
| 825.0 | 4.7912 | 929.1 | 1.000 |
| 850.0 | 4.7947 | 928.5 | 1.000 |
| 875.0 | 4.7980 | 927.8 | 1.000 |
| 900.0 | 4.8015 | 927.2 | 1.000 |
| 925.0 | 4.8048 | 926.6 | 1.000 |
| 950.0 | 4.8083 | 925.9 | 1.000 |
| 975.0 | 4.8115 | 925.3 | 1.000 |
| 1000.0 | 4.8148 | 925.1 | 1.000 |

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Case 3: Incompressible model - unconstrained

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| | | |
|---|---|---|
| TEMPERATURE AT LOWER ENTRANCE | -.0 | DEG C. |
| PRESSURE AT LOWER ENTRANCE | 950.000 | MB |
| PRESSURE AT UPPER ENTRANCE | 925.000 | MB |
| RELATIVE HUMIDITY AT LOWER ENTRANCE | 0.500 | |
| NUMBER OF DIVISIONS OF PASSAGE | 40 | |

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| | | |
|---|---|---|
| LENGTH OF PASSAGE | 1000.000 | M |
| RADIUS OF PASSAGE | 1.500 | M |
| ANGLE OF PASSAGE | 10.000 | DEG |
| WALL TEMPERATURE | 10.000 | DEG C. |
| FRICTION FACTOR | 0.030 | |
| FRACTION OF WALL THAT IS MOIST | 1.000 | |

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| DISTANCE (METRES) | TEMPERATURE (DEG CELSIUS) | PRESSURE (MILLIBARS) | DENSITY (KG/CUBIC M) |
|---|---|---|---|
| 0.0 | 0.0 | 949.9 | 1.2101 |
| 25.0 | -0.90 | 949.2 | 1.2128 |
| 50.0 | -0.84 | 948.6 | 1.2112 |
| 75.0 | -0.76 | 948.0 | 1.2095 |
| 100.0 | -0.46 | 947.3 | 1.2069 |
| 125.0 | -0.01 | 946.7 | 1.2038 |

| | | | |
|---|---|---|---|
| 150.0 | 0.54 | 946.1 | 1.2003 |
| 175.0 | 0.41 | 945.4 | 1.1998 |
| 200.0 | 1.07 | 944.8 | 1.1959 |
| 225.0 | 1.73 | 944.2 | 1.1920 |
| 250.0 | 2.40 | 943.5 | 1.1882 |
| 275.0 | 3.04 | 942.9 | 1.1845 |
| 300.0 | 3.66 | 942.3 | 1.1810 |
| 325.0 | 4.24 | 941.6 | 1.1776 |
| 350.0 | 4.79 | 941.0 | 1.1744 |
| 375.0 | 5.30 | 940.4 | 1.1714 |
| 400.0 | 5.76 | 939.7 | 1.1686 |
| 425.0 | 6.19 | 939.1 | 1.1660 |
| 450.0 | 6.58 | 938.5 | 1.1636 |
| 475.0 | 6.93 | 937.8 | 1.1613 |
| 500.0 | 7.25 | 937.2 | 1.1592 |
| 525.0 | 7.53 | 936.6 | 1.1572 |
| 550.0 | 7.79 | 935.9 | 1.1553 |
| 575.0 | 8.02 | 935.3 | 1.1536 |
| 600.0 | 8.22 | 934.7 | 1.1520 |
| 625.0 | 8.41 | 934.0 | 1.1504 |
| 650.0 | 8.57 | 933.4 | 1.1490 |
| 675.0 | 8.71 | 932.8 | 1.1476 |
| 700.0 | 8.84 | 932.2 | 1.1463 |
| 725.0 | 8.95 | 931.5 | 1.1451 |
| 750.0 | 9.05 | 930.9 | 1.1439 |
| 775.0 | 9.13 | 930.3 | 1.1428 |
| 800.0 | 9.21 | 929.6 | 1.1417 |
| 825.0 | 9.28 | 929.0 | 1.1406 |
| 850.0 | 9.34 | 928.4 | 1.1396 |
| 875.0 | 9.39 | 927.7 | 1.1387 |
| 900.0 | 9.43 | 927.1 | 1.1377 |
| 925.0 | 9.47 | 926.5 | 1.1368 |
| 950.0 | 9.51 | 925.9 | 1.1359 |
| 975.0 | 9.54 | 925.2 | 1.1350 |
| 1000.0 | 9.57 | 925.0 | 1.1341 |

| DISTANCE (METRES) | VELOCITY (M/SEC) | STAGNATION PRESSURE (MILLIBARS) | RELATIVE HUMIDITY |
|---|---|---|---|
| 0.0 | 4.4700 | 950.0 | 0.500 |
| 25.0 | 4.4601 | 949.4 | 0.728 |
| 50.0 | 4.4662 | 948.7 | 0.994 |
| 75.0 | 4.4724 | 948.1 | 1.165 |
| 100.0 | 4.4819 | 947.5 | 1.306 |
| 125.0 | 4.4936 | 946.8 | 1.396 |
| 150.0 | 4.5066 | 946.2 | 1.446 |
| 175.0 | 4.5086 | 945.5 | 1.471 |
| 200.0 | 4.5232 | 944.9 | 1.554 |
| 225.0 | 4.5379 | 944.3 | 1.538 |
| 250.0 | 4.5525 | 943.6 | 1.511 |
| 275.0 | 4.5667 | 943.0 | 1.477 |
| 300.0 | 4.5804 | 942.4 | 1.440 |
| 325.0 | 4.5935 | 941.7 | 1.401 |

| | | | |
|---|---|---|---|
| 350.0 | 4.6059 | 941.1 | 1.364 |
| 375.0 | 4.6177 | 940.5 | 1.328 |
| 400.0 | 4.6288 | 939.8 | 1.294 |
| 425.0 | 4.6391 | 939.2 | 1.262 |
| 450.0 | 4.6489 | 938.6 | 1.234 |
| 475.0 | 4.6580 | 938.0 | 1.207 |
| 500.0 | 4.6665 | 937.3 | 1.184 |
| 525.0 | 4.6745 | 936.7 | 1.162 |
| 550.0 | 4.6820 | 936.1 | 1.143 |
| 575.0 | 4.6891 | 935.4 | 1.126 |
| 600.0 | 4.6957 | 934.8 | 1.110 |
| 625.0 | 4.7020 | 934.2 | 1.096 |
| 650.0 | 4.7079 | 933.5 | 1.084 |
| 675.0 | 4.7135 | 932.9 | 1.073 |
| 700.0 | 4.7188 | 932.3 | 1.064 |
| 725.0 | 4.7239 | 931.6 | 1.055 |
| 750.0 | 4.7288 | 931.0 | 1.047 |
| 775.0 | 4.7335 | 930.4 | 1.040 |
| 800.0 | 4.7380 | 929.8 | 1.034 |
| 825.0 | 4.7423 | 929.1 | 1.029 |
| 850.0 | 4.7465 | 928.5 | 1.024 |
| 875.0 | 4.7506 | 927.9 | 1.020 |
| 900.0 | 4.7546 | 927.2 | 1.016 |
| 925.0 | 4.7585 | 926.6 | 1.012 |
| 950.0 | 4.7623 | 926.0 | 1.009 |
| 975.0 | 4.7661 | 925.4 | 1.006 |
| 1000.0 | 4.7698 | 925.1 | 1.003 |

APPENDIX B: PROGRAMS

```
C
C
C ****************************************************************
C
C THE COMPRESSIBLE MODEL
C
C ****************************************************************
C
C
C THIS PROGRAM CALCULATES SPELEOMICROCLIMATIC PARAMETRES
C GIVEN CERTAIN CHARACTERISTICS OF THE CAVE AND THE
C EXTERNAL ATMOSPHERIC CONDITIONS.  THE AIR IS ASSUMED
C TO BE COMPRESSIBLE.
C
C
C
      DOUBLE PRECISION M,SVP,MTEST,DABS,ESTMTD,ACTUAL
      DIMENSION T(101),TO(101),TV(101),D(101),DO(101)
      DIMENSION P(101),PO(101),QSPEC(101)
      DIMENSION M(101),V(101),RHMD(101)
      DIMENSION FI(101),FII(101),FIII(101)
      DIMENSION X(101)
      REAL L,LENGTH,K,JI,JII,LAPSE,INPUT
C
C
C VIRT(A,B) CALCULATES THE VIRTUAL TEMPERATURE AT
C TEMPERATURE A AND SPECIFIC HUMIDITY B
C
C
      VIRT(A,B)=A*(1.0+0.609*B)
C
C
C READ THE EXTERNAL ATMOSPHERIC CONDITIONS
C
C TEXTL  = ATMOSPHERIC TEMPERATURE AT LOWER ENTRANCE
C PEXTL  = ATMOSPHERIC PRESSURE AT LOWER ENTRANCE
C PEXTU  = ATMOSPHERIC PRESSURE AT UPPER ENTRANCE
C RH     = RELATIVE HUMIDITY AT LOWER ENTRANCE (DECIMAL)
C N      = NUMBER OF DIVISIONS AT WHICH PARAMETRES ARE
C          CALCULATED (THE LARGER N IS, THE GREATER THE
C          ACCURACY; MAXIMUM = 100)
C
C
      READ(4,100) TEXTL,PEXTL,PEXTU,RH,N
  100 FORMAT(F6.2,T10,F6.1,T20,F6.1,T30,F4.1,T40,I4)
C
C
C READ THE CHARACTERISTICS OF THE CAVE
C
C LENGTH = LENGTH OF CAVE
C FCOEFF = COEFFICIENT OF FRICTION (FANNING)
```

```
C RADIUS = RADIUS OF PASSAGE
C TWALL  = WALL TEMPERATURE IN CAVE
C THETA  = ANGLE OF PASSAGE
C E      = FRACTION OF PASSAGE WALL THAT IS MOIST (0.0 OR
C           1.0)
C
C
      READ(4,101) LENGTH,FCOEFF,RADIUS,TWALL,THETA,E
  101 FORMAT(F7.1,T10,F5.3,T20,F5.2,T30,F6.2,T40,F4.1,T50
     1       ,F3.1)
C
C
C CONSTANTS
C
C K      = RATIO OF SPECIFIC HEATS FOR DRY AIR
C TFREEZ = FREEZING POINT OF WATER
C RGAS   = GAS CONSTANT FOR DRY AIR
C RVAP   = GAS CONSTANT FOR WATER VAPOR
C CP     = SPECIFIC HEAT CONSTANT PRESSURE
C G      = ACCELERATION DUE TO GRAVITY
C L      = DRY ADIABATIC LAPSE RATE
C
C
      K      = 1.4
      TFREEZ = 273.16
      RGAS   = 287.04
      RVAP   = 461.50
      CP     = 1004.0
      G      = 9.80
      L      = 0.00976
C
C
C PO(1) AND TO(1) ARE THE INITIAL STAGNATION PRESSURE AND
C TEMPERATURE
C
C X(I) IS THE DISTANCE INTO THE CAVE, WHILE DELTA IS THE
C INCREMENT OF DISTANCE BETWEEN CALCULATED CROSS-SECTIONS
C
C
      PEXTL=100.*PEXTL
      PEXTU=100.*PEXTU
      PO(1)=PEXTL
      TO(1)=TEXTL+TFREEZ
      XN=N
      NN=N+1
      DELTA=LENGTH/(XN)
      X(1)=0.0
      DO 102 I=2,NN
      X(I)=X(I-1)+DELTA
  102 CONTINUE
      TWALL=TWALL+TFREEZ
      SMOOTH=1.0
C
C
```

```
C QWALL     = SPECIFIC HUMIDITY AT SATURATION FOR THE
C             WALL TEMPERATURE AT ATMOSPHERIC PRESSURE
C QST       = SPECIFIC HUMIDITY AT SATURATION FOR
C             INITIAL ATMOSPHERIC CONDITIONS
C QSPEC(I)  = SPECIFIC HUMIDITY AT DISTANCE X(I) INTO CAVE
C T(I)      = TEMPERATURE AT X(I)
C TV(I)     = VIRTUAL TEMPERATURE AT X(I)
C D(I)      = AIR DENSITY AT X(I)
C W         = MASS RATE OF FLOW
C
C
      QWALL=SAT(TWALL,PO(1))
      QST=SAT(TO(1),PO(1))
      QSPEC(1)=(QST*RH)/((QST*(RH-1.0))+1.0)
C
C
C ASSUME INITIAL VELOCITY = 1.0 AND CALCULATE INITIAL
C CONDITIONS, ESPECIALLY THE INITIAL MACH NUMBER M(1)
C
C
      V(1)=1.0
  103 READ(5,999) INPUT
  999 FORMAT(F6.3)
      IF(INPUT .NE. 0.0) V(1)=INPUT
      T(1)=TO(1)+(V(1)**2.0)/(2.0*CP)
      TV(1)=VIRT(T(1),QSPEC(1))
      M(1)=V(1)*((K*RGAS*TV(1))**(-0.5))
      FI(1)=(M(1)**2.0)*(1.0+K*(M(1)**2.0))/
     1      (1.0-K*(M(1)**2.0))
      FII(1)=(M(1)**2.0)*(2.0*K*(M(1)**2.0))/
     1       (1.0-K*(M(1)**2.0))
      FIII(1)=(M(1)**2.0)*2.0/(1.0-K*(M(1)**2.0))
      P(1)=PO(1)*((1.0 +(K-1.0)*(M(1)**2.0)/2.0)**((K-1.0)
     1     /K))
      D(1)=P(1)/(RGAS*TV(1))
      W=D(1)*3.14159*(RADIUS**2.0)*V(1)
C
C
C MONITR = MONITORING DEVICE FOR HUMIDITY (IF 0, AIR
C          IS NOT YET SATURATED, IF 1, IT IS)
C
C
      MONITR = 0
C
C
C SMOOTH = COEFFICIENT OF FRICTION FOR SMOOTH PASSAGE
C XO     = RELAXATION LENGTH
C
C
      IF(FCOEFF.EQ.SMOOTH) FCOEFF=2.0
      SMOOTH=0.25*(ABS(0.86859*ALOG((35886.12*W/RADIUS)/
     1       (1.964*ALOG(35886.12*W/RADIUS-3.8215)))**
     2       (-2.0))
      IF(FCOEFF.GE.1.000) FCOEFF=SMOOTH
```

```
      XO=((SMOOTH/FCOEFF)**0.5)*169.86*(RADIUS**0.8)*
     1    (W**0.2)
C
C
C BEGIN DO LOOP TO CALCULATE SUBSEQUENT CROSS-SECTIONS
C
C
      DO 110 I=2,NN
C
C
C IF THE TEMPERATURE IS BELOW FREEZING, H IS THE LATENT
C HEAT OF SUBLIMATION.  IF IT IS ABOVE FREEZING, H IS
C THE LATENT HEAT OF VAPORISATION
C
C
      IF(T(I-1).LT.TFREEZ)H=2.50E6
      IF(T(I-1).GE.TFREEZ)H=2.83E6
C
C
C THE TEMPERATURE IS NOW CALCULATED.  THE PARTICULAR
C FORMULA USED DEPENDS ON WHETHER OR NOT THE AIR IS
C SATURATED
C
C
      IF(MONITR.NE.0)GO TO 105
      QSPEC(I)=QWALL+(QSPEC(1-QWALL)*EXP(-X(I)/XO)
      IF(QSPEC(I).GE.SAT(T(I-1),PO(1)))GO TO 104
      T(I)=TWALL-L*XO*SIN(THETA*3.14159/180.0)
     1      +(T(1-TWALL+L*XO*SIN(THETA*3.14159/180.0))*
     2      EXP(-X(I)/XO)+(H*E*(QSPEC(1-QWALL)*X(I)*
     3      EXP(-E*X(I)/XO))/(CP*XO)
      GO TO 107
  104 XSAT=X(I-1)
      TSAT=T(I-1)
      MONITR=1
  105 QSPEC(I)=SAT(T(I-1),PO(1))
      T(I)=T(I-1)
      JI=QWALL*L/(CP*RVAP*(TWALL**2.0))
      JII=L/(RVAP*(TWALL**2.0))
      LAPSE=L*XO*SIN(THETA*3.14159/180.0)
  106 TTEST=TWALL-((1.0+JI)*LAPSE/(JI*JII*LAPSE+1.0))+
     1       (TSAT-TWALL+((1.0+JI)*LAPSE/(JI*JII*LAPSE+
     2       1.0)))*EXP(((ABS(JI*JII*LAPSE+1.0)**2.0)*
     3       (XSAT-X(I)-(T(I-TSAT)*XO*JI*JII*(JI*JII*
     4       LAPSE+1.0))/(XO*(1.0+JI)))
      IF(ABS(T(I-TTEST).LT.0.01)GO TO 107
      T(I)=TTEST
      GO TO 106
C
C
C CALCULATE VIRTUAL TEMPERATURE AND MACH NUMBER
C
C
  107 TV(I)=VIRT(T(I),QSPEC(I))
```

```
      M(I)=M(I-1)
  108 FI(I)=(M(I)**2.0)*(1.0+K*(M(I)**2.0))/
     1 (1.0-K*(M(I)**2.0))
      FII(I)=(M(I)**2.0)*(2.0*K*(M(I)**2.0))/
     1 (1.0-K*(M(I)**2.0))
      FIII(I)=(M(I)**2.0)*2.0/(1.0-K*(M(I)**2.0))
      MTEST=((M(I-1)**2.0)+(((FI(I)/TV(I))+FI(I-1)/TV(I-1))
     1        *0.5*(TV(I-TV(I-1)))+(FCOEFF*(FII(I)+FII(I-1))
     2        *(X(I-X(I-1))/(2.0*RADIUS))+(G*SIN(THETA*
     3        2.0*3.14159/360.0)*((FIII(I)/TV(I))+
     4        FIII(I-1)/TV(I-1))*(X(I-X(I-1))/(2.0*
     5        RGAS)))**(0.5)
      IF(DABS(M(I-MTEST).LT.(1.E-6))GO TO 109
      M(I)=MTEST
      GO TO 108
  109 P(I)=P(1)*(M(1)/M(I))*((TV(I)/TV(1))**0.5)
  110 CONTINUE
      WRITE(6,9000) V(1),P(N+1)
 9000 FORMAT(1X,F7.4,5X,F9.2)
C
C
C NOW ITERATE FOR INITIAL VELOCITY
C
C
      ESTMTD=DBLE(P(N+1))
      ACTUAL=DBLE(PEXTU)
      VTEST=V(1)*((ESTMTD/ACTUAL)**4.0)
      IF(ABS(V(1-VTEST).LT.0.0001)GO TO 111
      V(1)=VTEST
      GO TO 103
C
C
C CALCULATE ALL REMAINING PARAMETRES
C
C
  111 RHMD(1)=QSPEC(1)*(SAT(T(1),PO(1)-1.0)/
     1          (SAT(T(1),PO(1))*(QSPEC(1-1.0))
      DO 112 I=2,NN
  112 RHMD(I)=QSPEC(I)*(SAT(T(I-1),PO(1)-1.0)/
     1          (SAT(T(I-1),PO(1))*(QSPEC(I-1.0))
      DO 113 I=1,NN
      PO(I)=P(I)*((1.0+(K-1.0)*(M(I)**2.0)/2.0)**
     1       (K/(K-1.0)))
      D(I)=P(I)/(RGAS*TV(I))
      DO(I)=D(I)*((1.0+(K-1.0)*(M(I)**2.0)/2.0)**(1.0/
     1       (K-1.0)))
      TO(I)=T(I)*(1.0+(K-1.0)*(M(I)**2.0)/2.0-TFREEZ
      T(I)=T(I-TFREEZ
      TV(I)=TV(I-TFREEZ
      PO(I)=.01*PO(I)
      P(I)=.01*P(I)
      V(I)=V(1)*D(1)/D(I)
  113 CONTINUE
      TWALL=TWALL-TFREEZ
```

```
      PEXTU=.01*PEXTU
      PEXTL=.01*PEXTL
      WRITE(7,114)
  114 FORMAT('-')
      WRITE(7,115) TEXTL
  115 FORMAT(1X,'TEMPERATURE AT LOWER ENTRANCE',T40,
     1       F8.3,T50,'DEG C.')
      WRITE(7,116) PEXTL
  116 FORMAT(1X,'PRESSURE AT LOWER ENTRANCE',T40,
     1       F8.3,T50,'MB')
      WRITE(7,117) PEXTU
  117 FORMAT(1X,'PRESSURE AT UPPER ENTRANCE',T40,
     1       F8.3,T50,'MB')
      WRITE(7,118) RH
  118 FORMAT(1X,'RELATIVE HUMIDITY AT LOWER ENTRANCE',
     1       T40,F8.3)
      WRITE(7,119) N
  119 FORMAT(1X,'NUMBER OF DIVISIONS OF PASSAGE',T40,I4)
      WRITE(7,120)
  120 FORMAT('-')
      WRITE(7,121) LENGTH
  121 FORMAT(1X,'LENGTH OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,122) RADIUS
  122 FORMAT(1X,'RADIUS OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,123) THETA
  123 FORMAT(1X,'ANGLE OF PASSAGE',T40,F8.3,T50,'DEG')
      WRITE(7,124) TWALL
  124 FORMAT(1X,'WALL TEMPERATURE',T40,F8.3,T50,'DEG C.')
      WRITE(7,125) FCOEFF
  125 FORMAT(1X,'FRICTION FACTOR',T40,F8.3)
      WRITE(7,126) E
  126 FORMAT(1X,'FRACTION OF WALL THAT IS MOIST',T40,F8.3)
      WRITE(7,127)
  127 FORMAT('-')
      WRITE(7,128)
  128 FORMAT(1X,'DISTANCE',T15,'TEMPERATURE',T30,
     1       'PRESSURE',T45,'DENSITY')
      WRITE(7,129)
  129 FORMAT(1X,' (METRES) ',T15,' (DEG CELSIUS) ',T30,
     1       ' (MILLIBARS) ',T45,' (KG/CUBIC M) ')
      WRITE(7,130)
  130 FORMAT('-')
      DO 132 I=1,NN
      WRITE(7,131) X(I),T(I),P(I),D(I)
  131 FORMAT(1X,F7.1,T15,F6.2,T30,F6.1,T45,F6.4)
  132 CONTINUE
      WRITE(7,133)
  133 FORMAT('-')
      WRITE(7,134)
  134 FORMAT(1X,T15,'STAGNATION',T30,'STAGNATION',T45,
     1       'STAGNATION')
      WRITE(7,135)
  135 FORMAT(1X,'DISTANCE',T15,'TEMPERATURE',T30,
     1       'PRESSURE',T45,'DENSITY')
```

```
      WRITE(7,136)
  136 FORMAT(1X,'(METRES)',T15,'(DEG CELSIUS)',T30,
     1       '(MILLIBARS)',T45,'(KG/CUBIC M)')
      WRITE(7,137)
  137 FORMAT('-')
      DO 139 I=1,NN
      WRITE(7,138) X(I),TO(I),PO(I),DO(I)
  138 FORMAT(1X,F7.1,T15,F6.2,T30,F6.1,T45,F6.4)
  139 CONTINUE
      WRITE(7,140)
  140 FORMAT('-')
      WRITE(7,141)
  141 FORMAT(1X,'DISTANCE',T15,'VELOCITY',T30,'MACH',
     1       T45,'RELATIVE')
      WRITE(7,142)
  142 FORMAT(1X,'(METRES)',T15,'(M/SEC)',T30,'NUMBER',
     1       T45,'HUMIDITY')
      WRITE(7,143)
  143 FORMAT('-')
      DO 145 I=1,NN
      WRITE(7,144) X(I),V(I),M(I),RHMD(I)
  144 FORMAT(1X,F7.1,T15,F7.4,T30,F8.6,T45,F5.3)
  145 CONTINUE
      WRITE(7,146)
  146 FORMAT('-')
      STOP
      END
C
C
C SAT(A,B) CALCULATES THE SPECIFIC HUMIDITY AT SATURATION
C FOR TEMPERATURE A AND PRESSURE B
C
C SVP IS THE SATURATION VAPOR PRESSURE AT TEMPERATURE T
C
C
      FUNCTION SAT(A,B)
      IF(A.LT.273.16)GO TO 1000
      SVP =10.0**(-7.90298*((373.16/A-1.0)
     1      +5.02808*ALOG10(373.16/A)
     2      -(1.3816E-7)*(10.0**(11.344*(1.0-(A/373.16)))
     3      -1.0)+(8.1328E-3)*((10.0**(-3.49149*
     4      ((373.16/A-1.0))-1.0)+ALOG10(1013.246))
      GO TO 1001
 1000 SVP =10.0**(-9.09718*((273.16/A-1.0)
     1      -3.56654*ALOG10(273.16/A)
     2      +0.876793*(1.0-(A/273.16))+ALOG10(6.1071))
 1001 SAT=1.0-(1.0/(((0.62197*SVP)/(.01*B-SVP))+1.0))
      RETURN
      END
```

```
C
C
C ****************************************************************
C
C THE INCOMPRESSIBLE MODEL - CONSTRAINED
C
C ****************************************************************
C
C
C THIS PROGRAM CALCULATES SPELEOMICROCLIMATIC PARAMETRES
C GIVEN CERTAIN CHARACTERISTICS OF THE CAVE AND THE
C EXTERNAL ATMOSPHERIC CONDITIONS.  THE AIR IS ASSUMED
C TO BE INCOMPRESSIBLE.
C
C
C
      DIMENSION V(101),P(101),PO(101)
      DIMENSION T(101),D(101),TV(101)
      DIMENSION QSPEC(101),RHMD(101)
      DIMENSION X(101)
      REAL L,LENGTH,JI,JII,INTA,INTB,LAPSE,INPUT
C
C
C VIRT(A,B) CALCULATES THE VIRTUAL TEMPERATURE AT
C TEMPERATURE A AND SPECIFIC HUMIDITY B
C
C
      VIRT(A,B)=A*(1.0+0.609*B)
C
C
C READ THE EXTERNAL ATMOSPHERIC CONDITIONS
C
C TEXTL  = ATMOSPHERIC TEMPERATURE AT LOWER ENTRANCE
C PEXTL  = ATMOSPHERIC PRESSURE AT LOWER ENTRANCE
C PEXTU  = ATMOSPHERIC PRESSURE AT UPPER ENTRANCE
C RH     = RELATIVE HUMIDITY AT LOWER ENTRANCE
C N      = NUMBER OF DIVISIONS AT WHICH PARAMETRES ARE
C          CALCULATED (THE LARGER N IS, THE GREATER THE
C          ACCURACY; MAXIMUM = 100)
C
C
      READ(4,100) TEXTL,PEXTL,PEXTU,RH,N
  100 FORMAT(F6.2,T10,F6.1,T20,F6.1,T30,F4.1,T40,I4)
C
C
C READ THE CHARACTERISTICS OF THE CAVE
C
C LENGTH = LENGTH OF CAVE
C FCOEFF = COEFFICIENT OF FRICTION (FANNING)
C RADIUS = RADIUS OF PASSAGE
C TWALL  = WALL TEMPERATURE IN CAVE
C THETA  = ANGLE OF PASSAGE
C E      = FRACTION OF PASSAGE WALL THAT IS MOIST (0.0 OR
C          1.0)
```

```
C
C
      READ(4,101) LENGTH,FCOEFF,RADIUS,TWALL,THETA,E
  101 FORMAT(F7.1,T10,F5.3,T20,F5.2,T30,F6.2,T40,F4.1,T50
     1      ,F3.1)
C
C
C CONSTANTS
C
C TFREEZ = FREEZING POINT OF WATER
C RGAS   = GAS CONSTANT FOR DRY AIR
C RVAP   = GAS CONSTANT FOR WATER VAPOR
C CP     = SPECIFIC HEAT CONSTANT PRESSURE
C G      = ACCELERATION DUE TO GRAVITY
C L      = DRY ADIABATIC LAPSE RATE
C
C
      TFREEZ = 273.16
      RGAS   = 287.04
      RVAP   = 461.50
      CP     = 1004.64
      G      = 9.80
      L      = 0.00976
C
C
C PO(1) IS THE INITIAL STAGNATION PRESSURE
C
C X(I) IS THE DISTANCE INTO THE CAVE, WHILE DELTA IS THE
C INCREMENT OF DISTANCE BETWEEN CALCULATED CROSS-SECTIONS
C
C
      PEXTL=100.*PEXTL
      PEXTU=100.*PEXTU
      PO(1)=PEXTL
      XN=N
      NN=N+1
      DELTA=LENGTH/(XN)
      X(1)=0.0
      DO 102 I=2,NN
      X(I)=X(I-1)+DELTA
  102 CONTINUE
      TWALL=TWALL+TFREEZ
      SMOOTH=1.0
C
C
C QWALL    = SPECIFIC HUMIDITY AT SATURATION FOR THE
C            WALL TEMPERATURE
C QSPEC(I) = SPECIFIC HUMIDITY AT DISTANCE X(I) INTO CAVE
C QST      = SPECIFIC HUMIDITY AT SATURATION FOR
C            INITIAL ATMOSPHERIC CONDITIONS
C T(I)     = TEMPERATURE AT X(I)
C TV(I)    = VIRTUAL TEMPERATURE AT X(I)
C D(I)     = AIR DENSITY AT X(I)
C W        = MASS RATE OF FLOW
```

```
C
      T(1)=TEXTL+TFREEZ
      QWALL=SAT(TWALL,PO(1))
      QST=SAT(T(1),PO(1))
      QSPEC(1)=(QST*RH)/((QST*(RH-1.0))+1.0)
C
C
C CALCULATE THE VIRTUAL TEMPERATURE AND THE AIR
C DENSITY AT THE LOWER ENTRANCE
C
C
      TV(1)=VIRT(T(1),QSPEC(1))
      D(1)=PO(1)/(RGAS*TV(1))
C
C
C ASSUME INITIAL VELOCITY = 1.0 AND CALCULATE INITIAL
C PRESSURE
C
C
      V(1)=1.0
  103 READ(5,999) INPUT
  999 FORMAT(F6.3)
      IF(INPUT .NE. 0.0) V(1)=INPUT
      P(1)=PO(1-0.5*D(1)*(V(1)**2.0)
      W=D(1)*3.14159*(RADIUS**2.0)*V(1)
C
C
C MONITR = MONITORING DEVICE FOR HUMIDITY (IF 0, AIR
C          IS NOT YET SATURATED, IF 1, IT IS)
C
C
      MONITR = 0
C
C
C SMOOTH = COEFFICIENT OF FRICTION FOR SMOOTH PASSAGE
C XO     = RELAXATION LENGTH
C
C
      IF(FCOEFF.EQ.SMOOTH)FCOEFF=2.0
      SMOOTH=0.25*(ABS(0.86859*ALOG((35886.12*W/RADIUS)/
     1        (1.964*ALOG(35886.12*W/RADIUS-3.8215)))**
     2        (-2.0))
      IF(FCOEFF.GE.1.000)FCOEFF=SMOOTH
      XO=((SMOOTH/FCOEFF)**0.5)*169.86*(RADIUS**0.8)*
     1   (W**0.2)
C
C
C BEGIN DO LOOP TO CALCULATE SUBSEQUENT CROSS-SECTIONS
C
C
      DO 110 I=2,NN
C
C
C IF THE TEMPERATURE IS BELOW FREEZING, H IS THE LATENT
```

```
C HEAT OF SUBLIMATION.  IF IT IS ABOVE FREEZING, H IS
C THE LATENT HEAT OF VAPORISATION
C
C
      IF(T(I-1).LT.TFREEZ)H=2.50E6
      IF(T(I-1).GE.TFREEZ)H=2.83E6
C
C
C THE TEMPERATURE IS NOW CALCULATED.  THE PARTICULAR
C FORMULA USED DEPENDS ON WHETHER OR NOT THE AIR IS
C SATURATED
C
C
      IF(MONITR.NE.0)GO TO 105
      QSPEC(I)=QWALL+(QSPEC(1-QWALL)*EXP(-X(I)/XO)
      IF(QSPEC(I).GE.SAT(T(I-1),PO(1))) GO TO 104
      T(I)=TWALL-L*XO*SIN(THETA*3.14159/180.0)
     1     +(T(1-TWALL+L*XO*SIN(THETA*3.14159/180.0))*
     2     EXP(-X(I)/XO)+(H*E*(QSPEC(1-QWALL)*X(I)*
     3     EXP(-E*X(I)/XO))/(CP*XO)
      GO TO 107
  104 XSAT=X(I-1)
      TSAT=T(I-1)
      MONITR=1
  105 T(I)=T(I-1)
      QSPEC(I)=SAT(T(I-1),PO(1))
      JI=QWALL*L/(CP*RVAP*(TWALL**2.0))
      JII=L/(RVAP*(TWALL**2.0))
      LAPSE=L*XO*SIN(THETA*3.14159/180.0)
  106 TTEST=TWALL-((1.0+JI)*LAPSE/(JI*JII*LAPSE+1.0))+
     1     (TSAT-TWALL+((1.0+JI)*LAPSE/(JI*JII*LAPSE+
     2     1.0)))*EXP(((ABS(JI*JII*LAPSE+1.0)**2.0)*
     3     (XSAT-X(I)-(T(I-TSAT)*XO*JI*JII*(JI*JII*
     4     LAPSE+1.0))/(XO*(1.0+JI)))
      IF(ABS(T(I-TTEST).LT.0.01)GO TO 107
      T(I)=TTEST
      GO TO 106
C
C
C CALCULATE VIRTUAL TEMPERATURE
C
C
  107 TV(I)=VIRT(T(I),QSPEC(I))
C
C
C CALCULATE DENSITY
C
C
      D(I)=D(1)*(TV(1)/TV(I))*EXP((-G*SIN(THETA
     1     *3.14159/180.0)/RGAS)*((1.0/TV(I))+
     2     (1.0/TV(I-1)))*((X(I-X(I-1))/2.0))
     3     *(P(I-1)/P(1))
      P(I)=P(I-1-D(1)*(V(1)**2)*((D(1)/D(I)-1.0-
     1     G*SIN(THETA*3.14159/180.0)*(D(I)+D(I-1))*
```

```
     2      0.5*(X(I-X(I-1)-FCOEFF*(D(1)**2)*(V(1)
     3      **2)*((1.0/D(I))+(1.0/D(I-1)))*(X(I-X(I-1))/
     4      (2.0*RADIUS)
  110 CONTINUE
C
C
C NOW ITERATE FOR INITIAL VELOCITY
C
C
      INTA=D(1)+(4.0*D(2))+D(N+1)
      INTB=(1.0/D(1))+(4.0/D(2))+(1.0/D(N+1))
      DO 3000 I=4,N,2
      INTB=INTB+(2.0/D(I-1))+(4.0/D(I))
      INTA=INTA+(2.0*D(I-1))+(4.0*D(I))
 3000 CONTINUE
      P(N+1)=PEXTL-D(1)*((D(1)/D(N+1)-0.5)*(V(1)**
     1        2-G*SIN(THETA*3.14159/180.0)*INTA*
     2        DELTA/3.0-FCOEFF*(D(1)**2)*(V(1)**2)*
     3        INTB*DELTA/(3.0*RADIUS)
      VTEST=V(1)*((P(N+1)/PEXTU)**4)
      WRITE(6,2000) V(1),P(N+1)
 2000 FORMAT(1X,F7.4,5X,F8.2)
      IF(ABS(V(1-VTEST).LT.0.0001)GO TO 111
      V(1)=VTEST
      GO TO 103
C
C
C CALCULATE ALL REMAINING PARAMETRES
C
C
  111 RHMD(1)=QSPEC(1)*(SAT(T(1),PO(1)-1.0)/
     1         (SAT(T(1),PO(1))*(QSPEC(1-1.0))
      DO 112 I=2,NN
  112 RHMD(I)=QSPEC(I)*(SAT(T(I-1),PO(1)-1.0)/
     1         (SAT(T(I-1),PO(1))*(QSPEC(I-1.0))
      DO 113 I=1,NN
      V(I)=V(1)*D(1)/D(I)
      PO(I)=P(I)+0.5*D(I)*(V(I)**2.0)
      T(I)=T(I-TFREEZ
      TV(I)=TV(I-TFREEZ
      PO(I)=.01*PO(I)
      P(I)=.01*P(I)
  113 CONTINUE
      TWALL=TWALL-TFREEZ
      PEXTU=.01*PEXTU
      PEXTL=.01*PEXTL
      WRITE(7,114)
  114 FORMAT('-')
      WRITE(7,115) TEXTL
  115 FORMAT(1X,'TEMPERATURE AT LOWER ENTRANCE',T40,
     1      F8.3,T50,'DEG C.')
      WRITE(7,116) PEXTL
  116 FORMAT(1X,'PRESSURE AT LOWER ENTRANCE',T40,
     1      F8.3,T50,'MB')
```

```
      WRITE(7,117) PEXTU
117   FORMAT(1X,'PRESSURE AT UPPER ENTRANCE',T40,
     1      F8.3,T50,'MB')
      WRITE(7,118) RH
118   FORMAT(1X,'RELATIVE HUMIDITY AT LOWER ENTRANCE',
     1      T40,F8.3)
      WRITE(7,119) N
119   FORMAT(1X,'NUMBER OF DIVISIONS OF PASSAGE',T40,I4)
      WRITE(7,120)
120   FORMAT('-')
      WRITE(7,121) LENGTH
121   FORMAT(1X,'LENGTH OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,122) RADIUS
122   FORMAT(1X,'RADIUS OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,123) THETA
123   FORMAT(1X,'ANGLE OF PASSAGE',T40,F8.3,T50,'DEG')
      WRITE(7,124) TWALL
124   FORMAT(1X,'WALL TEMPERATURE',T40,F8.3,T50,'DEG C.')
      WRITE(7,125) FCOEFF
125   FORMAT(1X,'FRICTION FACTOR',T40,F8.3)
      WRITE(7,126) E
126   FORMAT(1X,'FRACTION OF WALL THAT IS MOIST',T40,F8.3)
      WRITE(7,127)
127   FORMAT('-')
      WRITE(7,128)
128   FORMAT(1X,'DISTANCE',T15,'TEMPERATURE',T30,
     1      'PRESSURE',T45,'DENSITY')
      WRITE(7,129)
129   FORMAT(1X,'(METRES)',T15,'(DEG CELSIUS)',T30,
     1      '(MILLIBARS)',T45,'(KG/CUBIC M)')
      WRITE(7,130)
130   FORMAT('-')
      DO 132 I=1,NN
      WRITE(7,131) X(I),T(I),P(I),D(I)
131   FORMAT(1X,F7.1,T15,F6.2,T30,F6.1,T45,F6.4)
132   CONTINUE
      WRITE(7,133)
133   FORMAT('-')
      WRITE(7,134)
134   FORMAT(1X,T30,'STAGNATION')
      WRITE(7,135)
135   FORMAT(1X,'DISTANCE',T15,'VELOCITY',T30,
     1      'PRESSURE',T45,'RELATIVE')
      WRITE(7,136)
136   FORMAT(1X,'(METRES)',T15,'(M/SEC)',T30,
     1      '(MILLIBARS)',T45,'HUMIDITY')
      WRITE(7,137)
137   FORMAT('-')
      DO 139 I=1,NN
      WRITE(7,138) X(I),V(I),PO(I),RHMD(I)
138   FORMAT(1X,F7.1,T15,F7.4,T30,F6.1,T45,F5.3)
139   CONTINUE
      WRITE(7,140)
140   FORMAT('-')
```

```
      STOP
      END
C
C
C SAT(A,B) CALCULATES THE SPECIFIC HUMIDITY AT SATURATION
C FOR TEMPERATURE A AND PRESSURE B
C
C SVP IS THE SATURATION VAPOR PRESSURE AT TEMPERATURE T
C
C
      FUNCTION SAT(A,B)
      IF(A.LT.273.16) GO TO 1000
      SVP =10.0**(-7.90298*((373.16/A-1.0)
     1     +5.02808*ALOG10(373.16/A)
     2     -(1.3816E-7)*(10.0**(11.344*(1.0-(A/373.16)))
     3     -1.0)+(8.1328E-3)*((10.0**(-3.49149*
     4     ((373.16/A-1.0))-1.0)+ALOG10(1013.246))
      GO TO 1001
 1000 SVP =10.0**(-9.09718*((273.16/A-1.0)
     1     -3.56654*ALOG10(273.16/A)
     2     +0.876793*(1.0-(A/273.16))+ALOG10(6.1071))
 1001 SAT=1.0-(1.0/(((0.62197*SVP)/(.01*B-SVP))+1.0))
      RETURN
      END
C
C
C ***************************************************************
C
C THE INCOMPRESSIBLE MODEL - UNCONSTRAINED
C
C ***************************************************************
C
C
C THIS PROGRAM CALCULATES SPELEOMICROCLIMATIC PARAMETRES
C GIVEN CERTAIN CHARACTERISTICS OF THE CAVE AND THE
C EXTERNAL ATMOSPHERIC CONDITIONS.  THE AIR IS ASSUMED
C TO BE INCOMPRESSIBLE.
C
C
C
      DIMENSION V(101),P(101),PO(101)
      DIMENSION T(101),D(101),TV(101)
      DIMENSION QSPEC(101),QSAT(101),RHMD(101)
      DIMENSION X(101)
      REAL L,LENGTH,INTA,INTB,INPUT
C
C
C VIRT(A,B) CALCULATES THE VIRTUAL TEMPERATURE AT
C TEMPERATURE A AND SPECIFIC HUMIDITY B
C
C
      VIRT(A,B)=A*(1.0+0.609*B)
C
C
```

```
C READ THE EXTERNAL ATMOSPHERIC CONDITIONS
C
C TEXTL  = ATMOSPHERIC TEMPERATURE AT LOWER ENTRANCE
C PEXTL  = ATMOSPHERIC PRESSURE AT LOWER ENTRANCE
C PEXTU  = ATMOSPHERIC PRESSURE AT UPPER ENTRANCE
C RH     = RELATIVE HUMIDITY AT LOWER ENTRANCE
C N      = NUMBER OF DIVISIONS AT WHICH PARAMETRES ARE
C          CALCULATED (THE LARGER N IS, THE GREATER THE
C          ACCURACY; MAXIMUM = 100)
C
C
      READ(4,100) TEXTL,PEXTL,PEXTU,RH,N
  100 FORMAT(F6.2,T10,F6.1,T20,F6.1,T30,F4.1,T40,I4)
C
C
C READ THE CHARACTERISTICS OF THE CAVE
C
C LENGTH = LENGTH OF CAVE
C FCOEFF = COEFFICIENT OF FRICTION (FANNING)
C RADIUS = RADIUS OF PASSAGE
C TWALL  = WALL TEMPERATURE IN CAVE
C THETA  = ANGLE OF PASSAGE
C E      = FRACTION OF PASSAGE WALL THAT IS MOIST (0.0 OR
C          1.0)
C
C
      READ(4,101) LENGTH,FCOEFF,RADIUS,TWALL,THETA,E
  101 FORMAT(F7.1,T10,F5.3,T20,F5.2,T30,F6.2,T40,F4.1,T50
     1       ,F3.1)
C
C
C CONSTANTS
C
C TFREEZ = FREEZING POINT OF WATER
C RGAS   = GAS CONSTANT FOR DRY AIR
C CP     = SPECIFIC HEAT CONSTANT PRESSURE
C G      = ACCELERATION DUE TO GRAVITY
C L      = DRY ADIABATIC LAPSE RATE
C
C
      TFREEZ = 273.16
      RGAS   = 287.04
      RVAP   = 461.50
      CP     = 1004.64
      G      = 9.80
      L      = 0.00976
C
C
C PO(1) IS THE INITIAL STAGNATION PRESSURE
C
C X(I) IS THE DISTANCE INTO THE CAVE, WHILE DELTA IS THE
C INCREMENT OF DISTANCE BETWEEN CALCULATED CROSS-SECTIONS
C
C
```

```
      PEXTL=100.*PEXTL
      PEXTU=100.*PEXTU
      PO(1)=PEXTL
      XN=N
      NN=N+1
      DELTA=LENGTH/(XN)
      X(1)=0.0
      DO 102 I=2,NN
      X(I)=X(I-1)+DELTA
  102 CONTINUE
      TWALL=TWALL+TFREEZ
      SMOOTH=1.0
C
C
C QWALL     = SPECIFIC HUMIDITY AT SATURATION FOR THE
C             WALL TEMPERATURE AT ATMOSPHERIC PRESSURE
C QSPEC(I)  = SPECIFIC HUMIDITY AT DISTANCE X(I) INTO CAVE
C QST       = SPECIFIC HUMIDITY AT SATURATION FOR
C             INITIAL ATMOSPHERIC CONDITIONS
C T(I)      = TEMPERATURE AT X(I)
C TV(I)     = VIRTUAL TEMPERATURE AT X(I)
C D(I)      = AIR DENSITY AT X(I)
C W         = MASS RATE OF FLOW
C
      T(1)=TEXTL+TFREEZ
      QWALL=SAT(TWALL,PO(1))
      QST=SAT(T(1),PO(1))
      QSPEC(1)=(QST*RH)/((QST*(RH-1.0))+1.0)
C
C
C CALCULATE THE VIRTUAL TEMPERATURE AND THE AIR
C DENSITY AT THE LOWER ENTRANCE
C
C
      TV(1)=VIRT(T(1),QSPEC(1))
      D(1)=PC(1)/(RGAS*TV(1))
C
C
C ASSUME INITIAL VELOCITY = 1.0 AND CALCULATE INITIAL
C PRESSURE
C
C
      V(1)=1.0
  103 READ(5,999) INPUT
  999 FORMAT(F6.3)
      IF(INPUT .NE. 0.0)V(1)=INPUT
      P(1)=PC(1-0.5*D(1)*(V(1)**2.0)
      QSAT(1)=SAT(T(1),PO(1))
      W=D(1)*3.14159*(RADIUS**2.0)*V(1)
C
C
C SMOOTH = COEFFICIENT OF FRICTION FOR SMOOTH PASSAGE
C XO     = RELAXATION LENGTH
C
```

```
C
      IF(FCOEFF.EQ.SMOOTH) FCOEFF=2.0
      SMOOTH=0.25*(ABS(0.86859*ALOG((35886.12*W/RADIUS)/
     1        (1.964*ALOG(35886.12*W/RADIUS-3.8215)))**
     2        (-2.0))
      IF(FCOEFF.GE.1.000) FCOEFF=SMOOTH
      XO=((SMOOTH/FCOEFF)**0.5)*169.86*(RADIUS**0.8)*
     1   (W**0.2)
C
C
C BEGIN DO LOOP TO CALCULATE SUBSEQUENT CROSS-SECTIONS
C
C
      DO 110 I=2,NN
C
C
C IF THE TEMPERATURE IS BELOW FREEZING, H IS THE LATENT
C HEAT OF SUBLIMATION.  IF IT IS ABOVE FREEZING, H IS
C THE LATENT HEAT OF VAPORISATION
C
C
      IF(T(I-1).LT.TFREEZ)H=2.50E6
      IF(T(I-1).GE.TFREEZ)H=2.83E6
C
C
C THE TEMPERATURE IS NOW CALCULATED
C
C
      QSPEC(I)=QWALL+(QSPEC(1-QWALL)*EXP(-X(I)/XO)
      L=0.00976
      T(I)=TWALL-L*XO*SIN(THETA*3.14159/180.0)
     1     +(T(1-TWALL+L*XO*SIN(THETA*3.14159/180.0))*
     2     EXP(-X(I)/XO)+(H*E*(QSPEC(1-QWALL)*X(I)*
     3     EXP(-E*X(I)/XO))/(CP*XO)
C
C
C CALCULATE VIRTUAL TEMPERATURE
C
C
  107 TV(I)=VIRT(T(I),QSPEC(I))
C
C
C CALCULATE DENSITY
C
C
      D(I)=D(1)*(TV(1)/TV(I))*EXP((-G*SIN(THETA
     1     *3.14159/180.0)/RGAS)*((1.0/TV(I))+
     2     (1.0/TV(I-1)))*((X(I-X(I-1))/2.0))
     3     *(P(I-1)/P(1))
      P(I)=P(I-1-D(1)*(V(1)**2)*((D(1)/D(I)-1.0-
     1     G*SIN(THETA*3.14159/180.0)*(D(I)+D(I-1))*
     2     0.5*(X(I-X(I-1)-FCOEFF*(D(1)**2)*(V(1)
     3     **2)*((1.0/D(I))+(1.0/D(I-1)))*(X(I-X(I-1))/
     4     (2.0*RADIUS)
```

```
  110 CONTINUE
C
C
C NOW ITERATE FOR INITIAL VELOCITY
C
C
      INTA=D(1)+(4.0*D(2))+D(N+1)
      INTB=(1.0/D(1))+(4.0/D(2))+(1.0/D(N+1))
      DO 3000 I=4,N,2
      INTB=INTB+(2.0/D(I-1))+(4.0/D(I))
      INTA=INTA+(2.0*D(I-1))+(4.0*D(I))
 3000 CONTINUE
      P(N+1)=PEXTL-D(1)*((D(1)/D(N+1)-0.5)*(V(1)**
     1        2-G*SIN(THETA*3.14159/180.0)*INTA*
     2        DELTA/3.0-FCOEFF*(D(1)**2)*(V(1)**2)*
     3        INTB*DELTA/(3.0*RADIUS)
      VTEST=V(1)*((P(N+1)/PEXTU)**4)
      WRITE(6,2000) V(1),P(N+1)
 2000 FORMAT(1X,F7.4,5X,F8.2)
      IF(ABS(V(1-VTEST).LT.0.0001)GO TO 111
      V(1)=VTEST
      GO TO 103
C
C
C CALCULATE ALL REMAINING PARAMETRES
C
C
  111 RHMD(1)=QSPEC(1)*(SAT(T(1),P(1)-1.0)/
     1         (SAT(T(1),P(1))*(QSPEC(1-1.0))
      DO 112 I=2,NN
  112 RHMD(I)=QSPEC(I)*(SAT(T(I-1),P(I-1)-1.0)/
     1         (SAT(T(I-1),P(I-1))*(QSPEC(I-1.0))
      DO 113 I=1,NN
      V(I)=V(1)*D(1)/D(I)
      PO(I)=P(I)+0.5*D(I)*(V(I)**2.0)
      T(I)=T(I-TFREEZ
      TV(I)=TV(I-TFREEZ
      PO(I)=.01*PO(I)
      P(I)=.01*P(I)
  113 CONTINUE
      TWALL=TWALL-TFREEZ
      PEXTU=.01*PEXTU
      PEXTL=.01*PEXTL
      WRITE(7,114)
  114 FORMAT('-')
      WRITE(7,115) TEXTL
  115 FORMAT(1X,'TEMPERATURE AT LOWER ENTRANCE',T40,
     1       F8.3,T50,'DEG C.')
      WRITE(7,116) PEXTL
  116 FORMAT(1X,'PRESSURE AT LOWER ENTRANCE',T40,
     1       F8.3,T50,'MB')
      WRITE(7,117) PEXTU
  117 FORMAT(1X,'PRESSURE AT UPPER ENTRANCE',T40,
     1       F8.3,T50,'MB')
```

```
      WRITE(7,118) RH
  118 FORMAT(1X,'RELATIVE HUMIDITY AT LOWER ENTRANCE',
     1      T40,F8.3)
      WRITE(7,119) N
  119 FORMAT(1X,'NUMBER OF DIVISIONS OF PASSAGE',T40,I4)
      WRITE(7,120)
  120 FORMAT('-')
      WRITE(7,121) LENGTH
  121 FORMAT(1X,'LENGTH OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,122) RADIUS
  122 FORMAT(1X,'RADIUS OF PASSAGE',T40,F8.3,T50,'M')
      WRITE(7,123) THETA
  123 FORMAT(1X,'ANGLE OF PASSAGE',T40,F8.3,T50,'DEG')
      WRITE(7,124) TWALL
  124 FORMAT(1X,'WALL TEMPERATURE',T40,F8.3,T50,'DEG C.')
      WRITE(7,125) FCOEFF
  125 FORMAT(1X,'FRICTION FACTOR',T40,F8.3)
      WRITE(7,126) E
  126 FORMAT(1X,'FRACTION OF WALL THAT IS MOIST',T40,F8.3)
      WRITE(7,127)
  127 FORMAT('-')
      WRITE(7,128)
  128 FORMAT(1X,'DISTANCE',T15,'TEMPERATURE',T30,
     1      'PRESSURE',T45,'DENSITY')
      WRITE(7,129)
  129 FORMAT(1X,'(METRES)',T15,'(DEG CELSIUS)',T30,
     1      '(MILLIBARS)',T45,'(KG/CUBIC M)')
      WRITE(7,130)
  130 FORMAT('-')
      DO 132 I=1,NN
      WRITE(7,131) X(I),T(I),P(I),D(I)
  131 FORMAT(1X,F7.1,T15,F6.2,T30,F6.1,T45,F6.4)
  132 CONTINUE
      WRITE(7,133)
  133 FORMAT('-')
      WRITE(7,134)
  134 FORMAT(1X,T30,'STAGNATION')
      WRITE(7,135)
  135 FORMAT(1X,'DISTANCE',T15,'VELOCITY',T30,
     1      'PRESSURE',T45,'RELATIVE')
      WRITE(7,136)
  136 FORMAT(1X,'(METRES)',T15,'(M/SEC)',T30,
     1      '(MILLIBARS)',T45,'HUMIDITY')
      WRITE(7,137)
  137 FORMAT('-')
      DO 139 I=1,NN
      WRITE(7,138) X(I),V(I),PO(I),RHMD(I)
  138 FORMAT(1X,F7.1,T15,F7.4,T30,F6.1,T45,F5.3)
  139 CONTINUE
      WRITE(7,140)
  140 FORMAT('-')
      STOP
      END
```